Math 402/502, Spring 2018
Assignment 9, due Wednesday, April 11

Problems to hand in:

1. Wade, Exercise 10.4.7.
2. Wade, Exercise 10.4.8.
3. Wade, Exercise 10.4.9.
4. Suppose (X, ρ) is a metric space and that $\{x_n\}$ is a convergent sequence with $\lim_{n\to\infty} x_n = a$. Use the definition of compactness to prove that the set
$$E = \{x_n : n \in \mathbb{N}\} \cup \{a\}$$
is compact. In other words, show directly that every open cover of E has a finite subcover.

   Note: E is just the union of the image of the sequence as a set in X with its limit $\{a\}$. Remark 10.15 may be helpful here.

On your own: Wade 10.4.1, 10.4.2, 10.4.3 as well as the following problems:

1. Prove that the following sets are not compact in $\mathbb{R}^n$ (with the usual metric) by exhibiting an open cover with no finite subcover. In particular, do not use the Heine-Borel theorem.
   (a) Any open ball $B_r(a)$ in $\mathbb{R}^n$.
   Hint: Fix y satisfying $|a - y| = r$ and consider the complement of closed balls of radius $1/n$ about y as a possible open cover of $B_r(a)$.
   (b) $\mathbb{N}$ as a subset of $\mathbb{R}$
   (c) $\{x \in \mathbb{Q} : 0 < x < 2\}$ as a subset of $\mathbb{R}$
2. Let $(\mathbb{R}, \sigma)$ be the discrete metric space from Example 10.3. Prove that $E \subset \mathbb{R}$ is compact if and only if it is finite.

Reading: Wade, sections 10.4, 10.5.