

Math 511, Spring 2018  
Assignment 9, Due Wednesday, April 11

Exercises to hand in:

1. The following are special cases of a “fact” concerning  $n$ -forms from class. Verify them rigorously in the following cases.

- (a) Let  $\omega = a_1 dx + a_2 dy$ ,  $\nu = b_1 dx + b_2 dy$  be 1-forms on  $T_p \mathbb{R}^2$ . Prove that there exists a constant  $c$  such that

$$\omega \wedge \nu = c dx \wedge dy,$$

identifying the constant  $c$  explicitly.

- (b) Now suppose that  $\omega$  is a 2-form on  $\mathbb{R}^3$ , that is  $\omega$  is a multilinear, alternating function  $\omega : T_p \mathbb{R}^3 \times T_p \mathbb{R}^3 \rightarrow \mathbb{R}$ . Show that

$$\omega = c_1 dx \wedge dy + c_2 dx \wedge dz + c_3 dy \wedge dz.$$

Again, identify the constants  $c_j$  explicitly in terms of the action of  $\omega$  on the standard basis vectors.

2. Show that if  $\omega = \alpha \wedge \beta$  is the wedge product of two 1-forms on  $T_p \mathbb{R}^3$ , then there is a line  $l$  in  $T_p \mathbb{R}^3$  such that if the plane spanned by  $V_1$  and  $V_2$  contains  $l$  then  $\omega(V_1, V_2) = 0$ .
3. Let  $\omega = dx + 5dy - dz$ ,  $\nu = 2dx - dy + dz$ ,  $\gamma = -dx + dy + 2dz$  denote 1-forms on  $T_p \mathbb{R}^3$ .

- (a) Find a constant  $c$  such that  $\omega \wedge \nu \wedge \gamma = c dx \wedge dy \wedge dz$ .
- (b) Let  $\alpha = 3dx \wedge dy + 2dy \wedge dz - dx \wedge dz$  be a 2-form on  $T_p \mathbb{R}^3$ . Find a constant  $c$  such that  $\alpha \wedge \gamma = c dx \wedge dy \wedge dz$ .

4. Suppose  $\omega$  is a 2-form on  $T_p \mathbb{R}^3$  such that

$$\omega = F_x dy \wedge dz - F_y dx \wedge dz + F_z dx \wedge dy.$$

Show that if  $V_\omega$  is the vector  $\langle F_x, F_y, F_z \rangle$ , then

$$\omega(V_1, V_2) = V_\omega \cdot (V_1 \times V_2)$$

where  $V_1 \times V_2$  is the usual cross product of  $V_1$  and  $V_2$  from multivariable calculus.

Note: Treat  $F_x, F_y, F_z$  as arbitrary scalars, not the partial derivatives of some function.

On your own: Exercises in Bachman (2nd Edition) 3.5-3.12, 3.14, 3.24, 3.33, 3.34, 3.36

Reading: Bachman (2nd Edition), Chapters 1-3, but don't spend much time on §3.5. Start chapter 4 for Week 12.