$\boxed{G(2,4)}$

$\begin{bmatrix} 1 & 0 & a_3 & a_4 \\ 0 & 1 & b_3 & b_4 \end{bmatrix} = U_{12}$; $\{U_{ij}\}$ covering.

(D) $G(2,4) = \{ 2 \text{ planes in } k^4 \} = \{ 2\times 4 \text{ mat's of rk } 2 \} / GL_2(k)$.

(D) Plücker emb $G(2,4) \to G(1,6) = \mathbb{P}^5$, $\Pi \subset V = k^4 \mapsto \overbrace{\Lambda^2 \Pi}^{\text{line}} \subset \Lambda^2 V = k^6$ (+)

Let $e_1, \dots, e_4$ can basis of $k^4 = V$ so $\{ e_i \wedge e_j \mid i<j \}$ basis of $\Lambda^2 V$

If $\Pi = kv + kw \Rightarrow \Lambda^2 \Pi = k \cdot v\wedge w$.

$v = \sum a_i e_i$
$w = \sum b_i e_i$ $\qquad v \wedge w = \sum_{i<j} (a_i b_j - a_j b_i) e_i \wedge e_j$ (+)

if $u = \sum p_{ij} e_i \wedge e_j$

($\mathrm{ch} \neq 2$) (L) Let $u \in \Lambda^2 V$. We have $u = v \wedge w$ for $v, w \in V \iff u \wedge u = 0 \iff p_{12}p_{34} - p_{13}p_{24} + p_{14}p_{23} = 0$ (*)

Pf. "$\Rightarrow$" $u \wedge u = v\wedge w \wedge v \wedge w = -v \wedge v \wedge w \wedge w = 0$.

"$\Leftarrow$" May assume $p_{12} = 1$. If rel (*) ok need to solve system $b_3 = p_{13}$, $b_4 = p_{14}$, $a_3 = -p_{23}$, $a_4 = -p_{24}$, $a_3 b_4 - a_4 b_3 = p_{34}$, All xcept last clear. Last is (*).

HOMOGENEOUS / NON-HOM POLS

$S^h = $ hom elts $\underbrace{}_{\text{of deg} \geq 1}$ in $S = k[x_0, \dots, x_n]$, $A = k[y_1, \dots, y_n]$

$\alpha : S^h \to A$, $\alpha(F) = F(1, y_1, \dots, y_n)$; $\beta : A \setminus k \to S^h$, $\beta(f) = x_0^{\deg f} f(\frac{x_1}{x_0}, \dots)$

α & β inv to each other on $S^h \setminus x_0 S^h$ & $A \setminus k$

(L) $f \in A$ irr $\Rightarrow \beta(f)$ irr $\quad [$ If $\beta(f) = GH$, $G, H \notin k$ $\Rightarrow f = \alpha\beta(f) = \alpha(G)\alpha(H)$ $\not\!\!\!\!\!\!\!\!\;\;\;$ in A, $x_0 \nmid G, H$ b/c $x_0 \nmid \beta(f)$ $]$

(P) $f \in A$ irr $\Rightarrow$ Zar cl of $Z(f)$ in $\mathbb{P}^n$ is $Z(\beta(f))$. $[\, Z(\beta(f)) \cap \mathbb{A}^n = \underbrace{Z(f)}_{\text{open in irr } Z(\beta(f))}$. $\forall$ open in irr sp is dense ! $]$

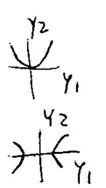
(C) $f \in A \setminus k \Rightarrow$ same. [EX].

(R) Zar cl of $Z(\underbrace{y_2 - y_1^2}_{f}, \underbrace{y_3 - y_1 y_2}_{g})$ is not $Z(\beta(f), \beta(g))$ [b/c latter is tw cubic $\cup$ line $\neq$ tw cubic]

CLASS OF CONICS, CUBICS

(D) projective trans of $\mathbb{P}^n$, $PGL_n(k)$ proj equivalence

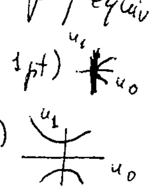
$f \in A$ irr conic $\Rightarrow \beta(f)$ irr conic $\overset{\text{s/lv}}{\Longrightarrow} \beta(f) \sim x_0^2 + x_1^2 + x_2^2 \Rightarrow$ all Zar cl of irr plane conics in $\mathbb{P}^2$ are proj equiv



(EX) $y_2 = y_1^2 \rightsquigarrow x_0 x_2 = x_1^2$. If $u_0 = \frac{x_0}{x_2}$, $u_1 = \frac{x_1}{x_2} \rightsquigarrow u_0 = u_1^2$ (meets line $u_0 = 0$ in 1 pt) @ ∞

$y_1^2 - y_2^2 = 1 \rightsquigarrow x_1^2 - x_2^2 = x_0^2$: —‖— $\rightsquigarrow u_1^2 - 1 = u_0^2$ (meets line $u_0 = 0$ in 2 pts) @ ∞

parab, hyp.



(+) $U_{12} \to U_{12} \cap \mathbb{P}^5$, $\begin{bmatrix} 1 & 0 & a_3 & a_4 \\ 0 & 1 & b_3 & b_4 \end{bmatrix} \mapsto e_1\wedge e_2 + b_3 e_1 \wedge e_3 + b_4 e_1 \wedge e_4 - a_3 e_2 \wedge e_3 - a_4 e_2 \wedge e_4 + (a_3 b_4 - a_4 b_3) e_3 \wedge e_4$
so injective.