Explicit computation of $H^1(\text{ell}, \mathcal{O})$

$E = U_1 \cup U_2$, $\quad U_1 = Z(y^2 - x(x-1)(x-\lambda))$, $\quad u = \dfrac{1}{y}$

$\qquad\qquad\qquad\quad U_2 = Z(u - v(v-u)(v-\lambda u))$, $\quad v = \dfrac{x}{y}$

$\mathcal{O}(U_1) = k[\bar{x}, \bar{y}]$, $\mathcal{O}(U_2) = k[\bar{u}, \bar{v}] = k[\bar{x}/\bar{y}, 1/\bar{y}]$

$\mathcal{O}(U_1 \cap U_2) = k[\bar{x}, \bar{y}, \bar{y}^{-1}] = \sum\limits_{n \in \mathbb{Z}} k\bar{y}^n + \sum\limits_{n \in \mathbb{Z}} k\bar{x}\bar{y}^n + \sum\limits_{n \in \mathbb{Z}} k\bar{x}^2\bar{y}^n.$

$\bar{y}^n \in \mathcal{O}(U_1)$, $n \geq 0$

$\bar{y}^n \in \mathcal{O}(U_2)$, $n < 0$

$\bar{x}\bar{y}^n \in \mathcal{O}(U_1)$, $n \geq 0$

$\bar{x}\bar{y}^n \in \mathcal{O}(U_2)$, $n < 0$

$\bar{x}^2\bar{y}^n \in \mathcal{O}(U_1)$, $n \geq 0$

$\bar{x}^2\bar{y}^n \in \mathcal{O}(U_2)$, $n < -1$

So $\forall$ elt in $\mathcal{O}(U_1 \cap U_2)$ is $\lambda\bar{x}^2/\bar{y} + a_2 - a_1$ w/ $a_i \in \mathcal{O}(U_i)$ So:

(P) $H^1(E, \mathcal{O}) = k \cdot [\bar{x}^2/\bar{y}]$ $\qquad$ (Pf. By $\ast$ $[\bar{x}^2/\bar{y}]$ generator; since $H^1(E,\mathcal{O})$ 1-dim $\Rightarrow$ $\ast$ is a basis)