$\boxed{SL_2(F)}$

F field, $U = \{ \underbrace{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}}_{u(b)} \in SL_2(F) \}$, $A = \{ \underbrace{\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}}_{s(a)} \in SL_2(F) \}$, U^t = transp of U

(L1) U & U^t generate $SL_2(F)$

Pf. Multip by $u(b)$ & $u(b)^t$ means eltary row & col ops. Can reduce any mat to $s(a)$. Now solve $\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}\begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ z & 1 \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$ for x, y, z, t
(4 equs, 4 unknowns)

(L2) $U^t = wUw^{-1}$, $w = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Pf. Clear

(T) $SL_2(F)$ is its own commutator if $\# F \geq 4$ (so $SL_2(F)$ not solvable)

Pf. $s(a)u(b)s(a)^{-1}u(b)^{-1} = u(ba^2 - b)$. Choose $a \in F$, $a^2 \neq 1$ & solve $ba^2 - b = x$. Get: any $u \in U$ is a commutator so $U \subset [G,G]$, $G = SL_2(F)$. Since $[G,G]$ normal $\Rightarrow$ $wUw^{-1} \subset [G,G]$. L1 & L2 $\Rightarrow$ $G = [G,G]$.

(T) $SL_2(F)/\text{center}$ is simple if $\# F \geq 4$

Pf. Needs more.