

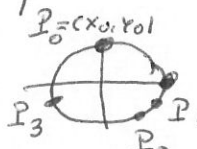
ADDITION FORMULAE

① $\sin(\alpha_1 + \alpha_2) = \sin \alpha_1 \cos \alpha_2 + \cos \alpha_1 \sin \alpha_2$
 $\alpha_i = \arcsin x_i, \quad y_i = \sqrt{1-x_i^2}; \quad \text{apply arcsin to}$

$\arcsin x_1 + \arcsin x_2 \equiv \arcsin(x_1 y_2 + x_2 y_1) \pmod{2\pi\mathbb{Z}}$

$$\int_{(x_0, y_0)}^{(x_1, y_1)} \frac{dx}{y} + \int_{(x_0, y_0)}^{(x_2, y_2)} \frac{dx}{y} \equiv \int_{(x_0, y_0)}^{(x_3, y_3)} \frac{dx}{y} \pmod{2\pi\mathbb{Z}}, \quad y^2 = 1-x^2$$

$P_0 = (x_0, y_0)$



genus 0

where $(x_0, y_0) = (0, 1), \quad (x_3, y_3) = (x_1, y_1) * (x_2, y_2)$

~~is a gr. iso.~~ i.e. $\int dx/y$ is a gr. iso. $SO(2, \mathbb{R}) \rightarrow \frac{\mathbb{R}}{2\pi\mathbb{Z}}$

$= (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$
 (addition * corresponds to adding angles $SO(2) = \left\{ \begin{pmatrix} x & y \\ -y & x \end{pmatrix} \mid x^2 + y^2 = 1 \right\}$)

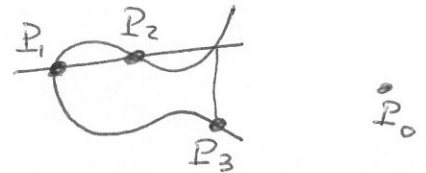
② (Fagnano-Euler)

$$\int_{(x_0, y_0)}^{(x_1, y_1)} \frac{dx}{y} + \int_{(x_0, y_0)}^{(x_2, y_2)} \frac{dx}{y} \equiv \int_{(x_0, y_0)}^{(x_3, y_3)} \frac{dx}{y} \pmod{\Lambda}, \quad y^2 = x^3 + ax + b$$

$(x_0, y_0) = \infty, \quad \Lambda = \text{"gr of periods"}$

i.e. $\int \frac{dx}{y}; E(\mathbb{C}) \rightarrow \frac{\mathbb{C}}{\Lambda}$ gr. iso

genus 1



(addition * is Δ)
 Group $(E, *)$

③ (Abel-Jacobi) higher genera.

$(\int w_1, \dots, \int w_g): C^0(X) \rightarrow \frac{\mathbb{C}^g}{\Lambda}$ gr. iso.