Areas & Volumes

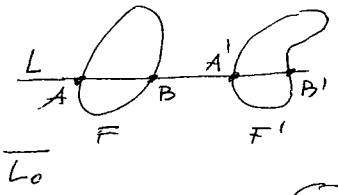
Area Axioms

0) Area () = $L \times W$

addit. princ → 1) Area $(F) = \text{Area}(F_1) + \text{Area}(F_2)$ if F decomposed in F_1, F_2

moving princ → 2) Area $(F) = \text{Area}(F')$ if F' obt from F by rigid motion

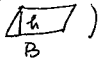
Cavalieri's principle 17AD = Archimedes' "method" → 3) If $\forall L \parallel L_0$



$AB \equiv A'B' \Rightarrow \text{Area}(F) = \text{Area}(F')$

don't need

$\Rightarrow$ 3') If $A'B' \equiv c\,AB \Rightarrow \text{Area}(F') = c\,\text{Area}(F)$

(T) Area () = $B \times h$

(T) Area () = $B \times h/2$

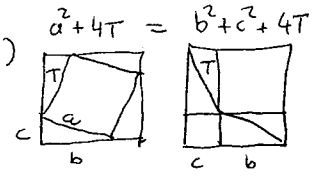
(T) Area () = $(B+b)h/2$

(T) Area () = perimeter $\times\, a/2$ $\Rightarrow$ Area ( ) = $\pi R^2/2$

reg pol

(D) $\pi = \frac{\text{circumf}}{\text{diam}}$

(T) (Pythagoras) 6BC

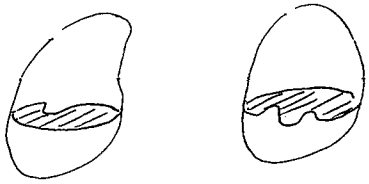


$\Rightarrow a^2 = b^2 + c^2$

Volume axioms

0) Vol () = abc

1) addit
2) moving princ
3) Cavalieri



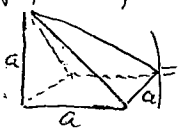
don't need

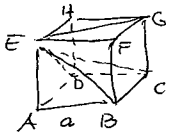
enhanced cavalieri

$\Rightarrow$ 3') analogous to 4

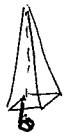
(T) Vol (parallelipiped), Vol (prism), Vol (cilinder)


(use Cavalieri)

(T) Vol (pyramid ) = $a^3/3$ [Pf



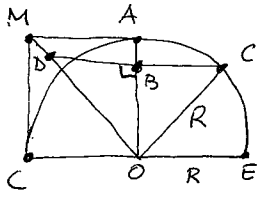
Vol(cube) = Vol(EABCD) + Vol(EBCGF) + Vol(EHGCD) = a^3 } equal volumes.

(T) Vol () = $\frac{b^2 h}{3}$

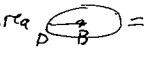
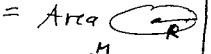
[Pf divide basis of into n^2 squares of side $\frac{a}{n}$ to do case $b = \frac{a}{n}$ & then do case $b = \frac{ma}{n}$]

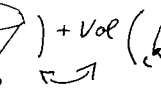
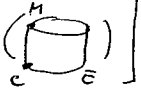
(T) Vol (cone) = $\frac{\pi R^2 h}{3}$

(T) Vol (sphere) = $\frac{4\pi R^3}{3}$ [Pf



$BC^2 = R^2 - OB^2 = R^2 - BD^2 \Rightarrow \pi BC^2 + \pi BD^2 = \pi R^2$

$\Rightarrow$ Area () + Area () = Area ()

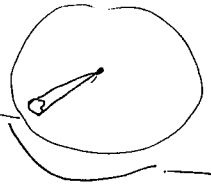
$\Rightarrow$ Vol () + Vol () = Area ()

(T) Area (sphere) = $4\pi R^2$]

Archimedes, "Method"

In Euclid too

[Pf: decompose sphere into "pyramids" (cones)



$\Rightarrow$ Vol = $\frac{R \times \text{Area}}{3}$

$\Rightarrow$ Area = ...]