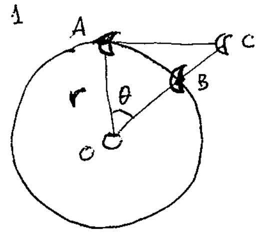
LAW OF UNIVERSAL GRAVITATION

Galileo: Rock(+) falling for t sec's travels distance(++) $d = \frac{gt^2}{2}$, $g = 9.81\, m/s^2 = a_\bullet$ (accel of rock $\bullet$)

Newton's hypothesis 1: The moon behaves like a falling rock(+) (acquiring some acceleration $a = a_☾$). After Δt sec's



$$\frac{a(\Delta t)^2}{2} \approx BC = CO - OB = \frac{r}{\cos\theta} - r = r\,\frac{1-\cos\theta}{\cos\theta}$$

$$= r\,\frac{1-\cos\frac{2\pi}{T}\Delta t}{\cos\frac{2\pi}{T}\Delta t} = 2r\,\frac{\sin^2\frac{\pi}{T}\Delta t}{\cos\frac{2\pi}{T}\Delta t}$$

$\frac{\theta}{2\pi} = \frac{\Delta t}{T}$, T = time of rev

$$\Rightarrow a \sim 4r\left(\frac{\sin\frac{\pi}{T}\Delta t}{\Delta t}\right)^2 \sim 4r\frac{\pi^2}{T^2}$$

($\cos 0 = 1$; $\frac{\sin ax}{x} \sim a$)

Same for planets (○) around Sun (☼)

Kepler's 3rd law: ○ = planet, ☼ = Sun

$$\frac{r_○^3}{T_○^2} = C_☼ \text{, a ct dep on ☼ but not on ○}$$

Cor $a_○ = 4r_○\frac{\pi^2}{T_○^2} = 4\frac{r_○^3}{r_○^2}\frac{\pi^2}{T_○^2} = \frac{4\pi^2 C_☼}{r_○^2}$ (more generally for any 2 bodies 1 & 2: $a_1 = 4\pi^2 C_2 / r_{12}^2$)

Newton's hyp 2: $a_☾ = \frac{4\pi^2 C_o}{r_☾^2}$, $a_\bullet = \frac{4\pi^2 C_o}{r_\bullet^2}$ (supp: analogy Sun | planets ; Earth | Moon (or rock))

Numerical verif: $9.81\, m/s^2 = a_\bullet \underset{hyp2}{=} \frac{4\pi^2 C_o}{r_\bullet^2} \underset{hyp2}{=} \frac{4\pi^2}{r_\bullet^2}\,\frac{a_☾ r_☾^2}{4\pi^2} \underset{hyp1}{=} \frac{1}{r_\bullet^2}\,4r_☾\frac{\pi^2}{T_☾^2}\cdot r_☾^2 \sim 9.81$ (dir calc (++ +))

Def (Newton): A phys sys is a set of fcns $\vec{x}_i : \mathbb{R}\to\mathbb{R}^3$ (trajectories), $\vec{F}_{ij} : \mathbb{R}\to\mathbb{R}^3$ (forces) & #'s $m_i \in \mathbb{R}$ (masses) s.t. $\vec F_{ij} + \vec F_{ji} = 0$ & $m_i\frac{d^2\vec x_i}{dt^2} = \vec F_i$, $\vec F_i = \sum_j \vec F_{ij}$.

Deduction of law of univ grav: 2 bodies $F_{ij} := |\vec F_{ij}|$ $\Rightarrow$ $F_{12} = F_{21}$

$$m_1\frac{4\pi^2 C_2}{r_{12}} \underset{hyp2}{=} m_1 a_1 = m_2 a_2 \underset{hyp2}{=} m_2\frac{4\pi^2 C_1}{r_{12}}$$

$\Rightarrow \frac{C_1}{m_1} = \frac{C_2}{m_2}$ for $\forall$ 2 bodies in universe; call this $\frac{k}{4\pi^2}$

$$\Rightarrow F_{12} = \frac{4\pi^2}{r_{12}^2}\, m_1 m_2 \frac{C_2}{m_2} = \frac{k\, m_1 m_2}{r_{12}^2}$$

(+) thrown horizontally [crossed out]

(++) a vertical

(++ +) $r_\bullet$ = rad of Earth, $r_☾$ = dis Earth to Moon, $T_☾$ = 28 days.