ⓡ quantities:
- math quant = $\mathbb{R}$ (can +, −, ·, ÷) = { 1, 2, 3, …, 0, −1, −2, …, $\frac{5}{6}$, $-\frac{3}{2}$, …, $\pi$, $\log 2$, $\sqrt{2}$, … } (#s) — nat; whole or int; rat; real
- physical q (can't + etc in gen) = { cm, gr, sec, c, $\hbar$, G, … } — **Planck units**

(EG) **Lengths** (distances): cm, in, km, mi (m), $d_{\oplus☾}$, $d_{\oplus☼}$, $R_{\oplus}$, $\ell$

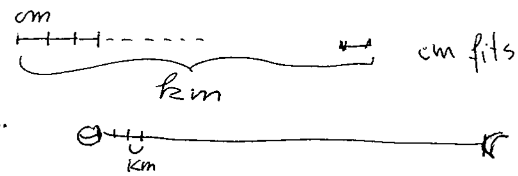
$d_{\oplus☾}$ = dist(Earth, Moon), $d_{\oplus☼}$ = dist(Earth, Sun), $R_\oplus$ = rad of Earth, $\ell$ = l of pencil

(EG) 1 km = 100,000 cm means: cm fits 100,000 times into km

$d_{\oplus☾}$ = 300,000 km means …. (How is this measured?)

= 300,000 × 100,000 cm

= $3 \cdot 10^{10}$ cm  (xpln $10^{10}$)



$d_1$ = ├─┤ , $d_2$ = ├─┼─┤ ⟹ $d_2 = 2d_1$, $d_1 = \frac{1}{2} d_2$

1 in = cm   $d_3$ = ├─┼─┼┤ ⟹ $d_3 = 2.5\, d_1$, $d_1 = \frac{1}{2.5} d_3$

1 mi = km

(F) 1) If $d_1, d_2$ lengths ∃ a (number) (xpln), $d_2 = a d_1$, $d_1 = \frac{1}{a} d_2$

2) —— ″ —— then $d_1 + d_2$ is a length (transf in cm); $c d_1$, $c d_2$ —″— (*)

(D) choice of a unit of length $\ell$; changing the unit, (EGs) $\ell_{int}$ = cm.

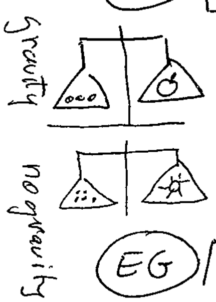
(EG) **Time** intervals (duration): sec, hour, year, t = time since Ramses II

hour = 3,600 sec

t = 3,200 yrs = …   (D) $t_{int}$ = sec

(F) Times can be added & mult by #'s (*)

(EG) **Masses** (weights): gr, lb, Kg, ton, $m_\oplus$, $m_☼$, $m_e$ (= mass of electron)




subtle difference; discuss weighing in outer space; mass conserved? relativity.

(EG) conversions

(F) masses can be + & mult by #'s (*)

(D) $m_{int}$ = gr

(EG) **Velocities** (speeds) (define): m/sec, mi/hour, c = speed of light = $3 \cdot 10^5$ km/sec = $3 \cdot 10^{10}$ cm/sec ($c_{int}$), $v_{sound}$

↑ upper lim in rel

(F) can be added & mult by #'s (*) (comp w/ ↗)

(*) L = {lengths} = vect sp of dim 1.   etc Later E = {energies} = $ML^2S^{-2}$ := $M \otimes L^{\otimes 2} \otimes (S)^{\otimes -2}$

T = {times} = —″—

(EG) **Accelerations** : $cm/sec^2$, $g = 9.81\ cm/sec^2$ (Galileo)

↑ define  (F) Form a vect sp.

(EG) **Force** : 1) gravitational $F = G\frac{mM}{d^2}$ (Newton), $\left(G = \underbrace{6.67\cdot 10^{-8}}_{G_{int}} \frac{cm^3}{gr\cdot sec^2}\right)$

↖ def as $ma$ (EG: $3\ gr\cdot cm\cdot sec^{-2}$)

2) electric $F = \frac{e_1 e_2}{d^2} = \frac{e^2}{d^2}$

3) strong (in nuclei)

4) weak (radiation)

(EG) **Energy** : kinetic $= \frac{mv^2}{2}$, potential $= mgh$, caloric, $5\ gr\cdot cm^2\cdot sec^{-2}$,

↑ Took a while to def & recogn that one can add these diff types. (19C)

(Planck, Einst.) $E_{photon\ (w/spc)} = \hbar\nu$  ($\nu$ = frequency $\overset{def}{=}$ # oscil/sec) QM

$\left(\hbar \simeq \underbrace{1.05\cdot 10^{-27}}_{\hbar_{int}}\ gr\cdot cm^2\cdot sec^{-1}\right)$

(Einst.) $E_{body\ @rest} = mc^2$  rel

→ (Q) Find units (Planck units) $\ell_p, m_p, t_p$ s.t. $c = 1\cdot \ell_p/t_p$ etc

↖ Planck length etc

Sol  Set $\ell_p = X\cdot cm$, $m_p = y\cdot gr$, $t_p = z\cdot sec$

$\Rightarrow c = c_{int}\ cm/sec = c_{int} X^{-1} z\ \ell_p/t_p \Rightarrow c_{int} X^{-1} z = 1$

$G = \ldots \Rightarrow G_{int} X^{-3} y z^2 = 1$

$\hbar = \ldots \Rightarrow \hbar_{int} X^{-2} y^{-1} z = 1$

$\Rightarrow X = c_{int}\cdot z \Rightarrow \begin{cases} G_{int} c_{int}^{-3} z^{-1} y = 1 \\ \hbar_{int} c_{int}^{-2} z^{-1} y^{-1} = 1 \end{cases} \Rightarrow G_{int}\hbar_{int} c_{int}^{-5} z^{-2} = 1$ (mult)

$\Rightarrow z = \sqrt{\frac{G_{int}\hbar_{int}}{c_{int}^5}} \Rightarrow X = \sqrt{\frac{G_{int}\hbar_{int}}{c_{int}^3}}$  $Y = \sqrt{\frac{G_{int}}{c_{int}\hbar_{int}}}$

$\ell_p \cong 1.61\cdot 10^{-33}\ cm$

$t_p \cong 5.39\cdot 10^{-44}\ sec$

} ridicul. small WHY? (smth wrong w/ universe or us?)

$m_p \approx 2.17\cdot 10^{-5}\ gr$ (like pharmacy)

(Q) $\frac{e^2}{4\pi}$ in Planck units is $\approx \frac{1}{137}$ (fine strct). WHY?

$4\pi\times$ (Force of attract b/t 2 electrons @ $1\cdot \ell_p$) $\approx \frac{1}{137}\frac{m_p \cdot \ell_p}{t_p^2}$

Planck's address / alien communic. imprinted his laws of phys → c

pure convention of humans

| | L | T | M |
|---|---|---|---|
| US unit sys | in | sec | ounce |
| internat | cm | sec | gr |
| Planck | $\ell_p$ | $t_p$ | $m_p$ |