Math 321

Linear Algebra

April 21, 2006 Quiz 10 Spring 2006

Name: Key — White

Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

a) Compute the inverse of A.

$\det A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} = 1 - (-2) = 3$

$\det A_{11} = |A_{11}| = 1 \quad |A_{12}| = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad |A_{13}| = -1 \quad |A_{21}| = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = 0$

$|A_{22}| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = 3 \quad |A_{23}| = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad |A_{31}| = \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} = 2$

$|A_{32}| = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 \quad |A_{33}| = 1 \Rightarrow$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 2 \\ 1 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

b) Solve the system of equations

$$\begin{aligned} x - 2z &= 1 \\ y + z &= 0 \\ x + z &= 2 \end{aligned}$$

This is $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

multiply by A^{-1} to give

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 2 \\ 1 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 5/3 \\ -1/3 \\ 1/3 \end{pmatrix}$$

or use Cramer's rule.

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