Math 321

Linear Algebra

March 31, 2006 Quiz 7 Spring 2006

Name: Key - White

1. Determine whether the linear map $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is invertible or not.

a) $L(x, y) = (y, x)$ $\ker L = \{(x,y) \mid L(x,y) = (0,0)\}$

$\Rightarrow (y,x) = (0,0) \Rightarrow y = 0 \quad x = 0 \Rightarrow \ker L = \{0\}$

So by a theorem L is invertible

b) $L(x, y) = (y, -y)$. Here $\ker L = \{(x,0)\} \neq \{0\}$

since $(y,-y) = (0,0) \Rightarrow y = 0$. So $\ker L \approx \mathbb{R}$

$\Rightarrow L$ is not injective $\Rightarrow L$ is not invertible

1. Let $P : V \longrightarrow V$ be a projection map, that is a linear map such that $P^2 = P$. Prove that the vector $v - P(v)$ is in the kernel of P.

$P(v - P(v)) = P(v) - P^2(v) = P(v) - P(v) = 0$

$\Rightarrow v - P(v) \in \ker P.$

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Name: Key - Yellow

1. Let $P : V \longrightarrow V$ be a projection map, that is a linear map such that $P^2 = P$. Prove that the vector $P(v) - v$ is in the kernel of P.

$P(P(v) - v) = P^2(v) - P(v) = P(v) - P(v) = 0$

$\Rightarrow P(v) - v \in \ker P$

2. Determine whether the linear map $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is invertible or not.

a) $L(x, y) = (x, x)$

$\ker L = \{(0, y)\}$ since $L(x, y) = 0 \Rightarrow x = 0$

Since $\ker L \neq \{0\}$ L is not invertible

b) $L(x, y) = (y, -x)$.

$\ker L(x, y) = \{(y, -x) = (0, 0)\} \Rightarrow x = 0 \quad y = 0$

$\Rightarrow \ker L = \{0\} \Rightarrow L$ is invertible