Math 321
Linear Algebra

April 6, 2006 | Quiz 8 | Spring 2006

Name: Key – Yellow

1. Give an orthonormal basis for the space generated by the vectors $(1, -1, -1)$ and $(2, 1, 0)$.

$v_1 = \frac{1}{\sqrt{3}}(1,-1,-1) \qquad v_2 = (2,1,0)$

$v_2' = \overset{(2,1,0)}{v_2} - \frac{(1,-1,1)(2,1,0)(1,-1,-1)}{3} = (2,1,0) - \frac{1}{3}(1,-1,-1)$

$= \frac{1}{3}(5,4,1)$. So $\perp$n basis is

$$\boxed{\frac{1}{\sqrt{3}}(1,-1,-1), \ \frac{1}{\sqrt{42}}(5,4,1)}$$

2. The trace of an $n \times n$ matrix $A = (a_{ij})$ is defined by $\mathrm{tr}(A) = \sum_i a_{ii}$. Show that the $\mathrm{tr}(AA^t)$ is positive where A^t is the transpose of A.

$tr(AA^t) = tr\left(\sum_{j=1}^{n} a_{ij}\, {}^t a_{jk}\right) = tr\left(\sum_{j=1}^{n} a_{ij} a_{kj}\right) = \sum_{i,j=1}^{n} a_{ij} a_{ij}$

$= \sum_{i,j=1}^{n} a_{ij}^2 > 0 \qquad A \neq 0$

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Name: Rey-White

1. The trace of an $n \times n$ matrix $A = (a_{ij})$ is defined by $\text{tr}(A) = \sum_i a_{ii}$. Show that the $\text{tr}(A^t A)$ is positive where A^t is the transpose of A.

$$tr(A^tA)_{ik} = tr\left(\sum_j a^t_{ij} a_{jk}\right) = tr\left(\sum_j a_{ji} a_{jk}\right)$$

$$= \sum_{j,i} a_{ji} a_{ji} = \sum_{j,i} a_{ji}^2 > 0 \qquad A \neq 0$$

2. Give an orthonormal basis for the space generated by the vectors $(1, 1, -1)$ and $(0, 2, 1)$.

$$\begin{array}{cc} \| & \| \\ V_1 & V_2 \end{array}$$

$$V_2' = V_2 - \frac{V_1 \cdot V_2}{\langle V_1, V_1 \rangle} V_1 = (0,2,1) - \frac{(1\ 1\ -1)\cdot(0\ 2\ 1)}{3}(1,1,-1)$$

$$= (0,2,1) - \frac{1}{3}(1,1,-1) = \frac{1}{3}(-1,5,4)$$

So $\perp n$ basis is $\|V_2'\|^2 = 25 + 16 + 1 = 42$

$$\boxed{\frac{1}{\sqrt{3}}(1,1,-1) \qquad \frac{1}{\sqrt{42}}(-1,5,4)}$$