Math 306 - College Geometry - Spring 2012

Group work 1 - follow up to class discussion on 1/23/12

1 Toy Model (modified slightly after our discussion yesterday) and hints for your model

In the Toy Model the plane $\mathcal{E}$ consists of *three different points*[1] on the Cartesian plane. To fix ideas we give names to the points, we can include coordinates, although in this case is really not necessary.

$$A = (x_1, y_1), \quad B = (x_2, y_2), \quad C = (x_3, y_3), \qquad \mathcal{E} = \{A, B, C\}.$$

In the Toy Model the set $\mathcal{L}$ of *lines* are any two-point subset of $\mathcal{E}$, namely:

$$\ell = \{A, B\}, \quad m = \{B, C\}, \quad n = \{A, C\}, \qquad \mathcal{L} = \big\{\ell, m, n\big\}.$$

In this particular model we can list the three points and the three lines. Notice that since the three points are different, the three lines are different lines as well.

> *In the models you are working on there are infinitely many lines, so you cannot list them all, instead you can describe them using auxiliary "parameters", and describing carefully the set of parameters.*

The Toy Model satisfies the two Incidence Axioms. Here is the proof.

1.1 First Incidence Axiom

We have to show that there are three non-collinear points in our Toy model plane $\mathcal{E}$. Since there are exactly three points in $\mathcal{E}$ we must show they do not belong to a line in our model. By inspection, we see that no line contains the three points, specifically: line ℓ does not contain point C, line m does not contain point A, and line n does not contain point B. Therefore the three points A, B and C are non-collinear in the Toy model plane $\mathcal{E}$.

> *In the models you are working on you have many more points at your disposal. You have to choose 3 specific points you think are good candidates to be non-collinear in your model plane, then you have*

[1] I wrote in the handout three non-collinear points in the Cartesian plane, which implies the points are different, and that is all we need.

to show that none of the lines in your model contain the three points. You should have good analytic description of the lines in your model (in particular the "parameters" describing them), and you can assume your three points are in a generic line (if there is more than one type of line, you have to do this for each type of line, this is the case for the Moulton Plane and the Poincaré Disk), manipulate your expressions and reach a contradiction. The conclusion will be the three points are not collinear.

1.2 Second Incidence Axiom

We have to show that given any two points in our model plane there is exactly one line in $\mathcal{L}$ containing them.

This is clear by inspection once more. We can list all sets of two points and the unique line they belong to. The points A and B belong to the line ℓ, the points B and C belong to the line m, and the points A and C belong to the line n. These are the only lines that contain each pair of points because: the line ℓ does not contain the point C, therefore it cannot contain the pairs of points B, C or A, C; the line m does not contain the point A therefore it cannot contain the pairs of points B, A or A, C, finally the line n does not contain the point B therefore it cannot contain the pairs of points B, A or B, C. The order in which the points are given doesn't matter, the points B and A belong to the line ℓ, etc. This is because the the lines are sets, and the order in which the elements are listed in the set doesn't matter.

In the models you are working on you have infinitely many points at your disposal. Therefore there are infinitely many sets of two different points. You cannot list them all. You will have to take two generic points in your model plane (here is important to have good understanding of what these points are so that you can manipulate them), and you will have to find a line in your model that contains them, in this case it means find appropriate parameters (that will depend on the two given points) that define one of your lines (again this is harder when you have more than one type of lines, because you have to consider cases, when the points are in one type of lines, you must show that they are not in the other types, furthermore you must ensure that there is exactly one line that contains both points (usually this will be a consequence of the unique set of parameters that you will associate to the given pair of points).