Department of Mathematics and Statistics University of New Mexico

Group Work 1 Math 510: Real Analysis 1 Fall 2020

Instructions: Find a group of 2-3 people and choose two of Problems A-E for 50 points. Turn in one write up per group. Due on Thursday Sep 24, 2020

- A. (Rudin Chapter 3 #19) Characterization of the Cantor set via ternary expansions.
- B. (Rudin Chapter 3 #16 and #17) Algorithms to find the square root of a positive number.
- C. (Rudin Chapter 3 # 24) Every metric space can be completed.
- **D.** (Qual Jan 1999 #1) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Let $r = \limsup_{n \to \infty} |a_n|^{\frac{1}{n}}$.
 - (a) Show that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely if r < 1 and diverges if r > 1. (Qual Jan 2012 #2)
 - (b) Show that the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is given by $R = r^{-1}$.
- **E.** (Qual Aug 2011 #4) Given a 2 dimensional vector (x_1, x_2) , define its *p*-norm as

$$\|(x_1, x_2)\|_p = \begin{cases} (|x_1|^p + |x_2|^p)^{1/p} & 1 \le p < \infty \\ \max(|x_1|, |x_2|) & p = \infty \end{cases}$$

(a) In the Euclidean plane, geometrically describe the "unit balls"

$$\{(x_1, x_2) : \|(x_1, x_2)\|_p \le 1\}$$

in the $p = 1, 2, \infty$ norms.

(b) Show that for a vector (x_1, x_2) its *p*-norm converges to its ∞ -norm as $p \to \infty$. In other words, show that

$$\lim_{p \to \infty} \|(x_1, x_2)\|_p = \|(x_1, x_2)\|_{\infty}.$$