University of New Mexico-Math 163: Calculus II

Spring 2019

Course Description

Transcendental functions, techniques of integration, numerical integration, improper integrals, sequences and series, Taylor series with applications, complex variables, differential equations.

Required Materials

- *Calculus*, Stewart, 7e

Prerequisites

Prerequisites: Math 162 (or equivalent) with ”C” (not ”C−”) or better

Course Objectives

1. Know the definitions, graphs, special values, derivatives and integrals (when possible) of transcendental functions, including exponential, logarithmic, inverse trigonometric and hyperbolic functions.
2. Use the methods of substitution, integration by parts, partial fractions and trigonometric substitution to compute proper and improper integrals. Evaluate improper integrals using correct mathematical limit notation.
3. Use rectangles or trapezoids to approximate integrals.
4. Solve separable differential equations. Plot direction fields and solution curves. Find equilibrium solutions.
5. State the definition of the value of a series, as well as necessary conditions for convergence. Use the definition to determine the value of a series. Determine the value of known Taylor series at particular points. State various tests for convergence, including all conditions, and apply them. Approximate alternating series and estimate the error.

6. Determine the asymptotic behaviour of functions $f(x)$ as $x \to \pm\infty$ and the limit of indeterminate forms.

7. State the definition of the Taylor series of a function and describe its properties. Find Taylor series using the definition, or by substitution into, or differentiation or integration of known series, and determine their interval/radius of convergence. Approximate functions by Taylor polynomials within the interval of convergence and estimate the error. Include approximations of definite integrals or quantities depending on parameters, such as arise in applications in physics, chemistry, biology and engineering.

8. Use Taylor series to derive Euler's formula for the exponential of a complex number. Evaluate sums, products, powers, roots, and exponentials of complex numbers. Evaluate integrals of complex functions.

Grading Policy

You grade will be based on the following:

- **3 in-class exams** (100 points each). Exams will be based on homework and examples discussed in class. No calculators, notes, or any kind of electronic device may be used on exams. There are **NO MAKEUP EXAMS UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**.

- **Written Homework** (50 points) HW is due at the beginning of class. e-mailed HW will not be accepted. Your written work should be neatly organized, showing all steps and using proper mathematical notation. Your papers must be stapled. In order to ensure that assignments are graded promptly, and to discourage students from falling behind, **LATE ASSIGNMENTS WILL NOT BE ACCEPTED UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**. However, in recognition of the fact that unavoidable issues sometimes arise, the lowest of each student's homework grades (including zeros for unsubmitted assignments) will be dropped when calculating final semester grades. Despite this policy, you should complete every assignment, even if you miss a deadline, because understanding the homework will help you perform well on exams.

- **Recitation** (100 points) Students will receive a grade for each recitation that will be based partially on attendance, and partially on quizzes, participation, group problem-solving, or any combination of these. See the Recitation Handout for more infomation. There will be 12 quizzes with the lowest score dropped. There are **NO MAKEUP QUIZZES UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**.

- **Final Exam** (200 points) The final exam will be cumulative and held on Monday, May 6th at 7:30 AM. **NO EARLY FINAL EXAMS NOR MAKE UP EXAMS WILL BE GIVEN UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**.

Course Policies

Student Behavior

All students must abide by the Student Code of Conduct: pathfinder.unm.edu. According to the Code of Conduct, student activities that interfere with the rights of others to pursue their education or to conduct their University duties and responsibilities will lead to disciplinary action. This includes any activities that are disruptive to the class and any acts of academic dishonesty. Students are expected to behave in a courteous and respectful manner toward the instructor and their fellow students. The use of cell phones, headphones, smart watches, etc. is not permitted during class or exams. .

Attendance Policy

Attendance is mandatory. If a student has four or more unexcused absences he/she may be dropped from the course. Tardiness or early departure may be regarded as an absence. After the Withdrawal deadline the instructor will not drop any student. Please note that it is the student's responsibility to drop the course if he/she stops attending. A failing grade of F may be assigned if the student stops attending and does not drop before the posted deadline. **No early final exams** will be given except in documented emergencies: flight reservations, weddings, vacations, birthdays, non-NCAA sporting events etc. are not considered emergencies.

Policies on Grading Options

You must select your grade mode (Letter Grade, CR/NC, or Audit) within the first 2 weeks of the semester. We will not give permission to change the grade mode after the deadline. Students who are in the regular grade mode and who withdraw after the end of week 3 will receive a grade of W. If you do not withdraw, you will receive a letter grade of A, B, C, D, or F (not a W). Students who are in the CR/NC grade mode and who withdraw after the end of week 3 will receive a grade of W. If you do not withdraw, you will receive a letter grade of NC (not a W). See the list of all deadlines: registrar.unm.edu.

Deadlines

The Department of Mathematics and Statistics will adhere to all of the registration deadlines published by the Office of the Registrar in the schedule of classes: registrar.unm.edu . We will not give permission to override any deadline except in documented emergencies.

Academic Integrity and Honesty

Cheating of any kind will not be tolerated. Examples are: looking at a neighbor's exam, plagiarizing, using a calculator when not permitted, using the book and/or a cheat sheet, modifying an exam after it is graded, etc. The instructor may warn an offending student, the score of the exam may be reduced, the score may be set to zero, the student may be dropped from the class, the student may get a grade of F for the class, and in most cases the incident will be reported to the Dean of Students..

Accommodations for Disabilities

We will accommodate students with documented disabilities through the Accessibility Resource Center (ARC). During the first two weeks of the semester, those students should inform the instructor of their particular needs.

Title IX

In an effort to meet obligations under Title IX, UNM faculty, Teaching Assistants, and Graduate Assistants are considered "responsible employees" by the Department of Education. This designation requires that any report of gender discrimination which includes sexual harassment, sexual misconduct and sexual violence made to a faculty member, TA, or GA must be reported to the Title IX Coordinator at the Office of Equal Opportunity, oeo.unm.edu. For more information on the university policy regarding sexual misconduct, see: https://policy.unm.edu/university-policies/2000/2740.html.

Schedule and Weekly Learning Goals

This schedule is tentative and subject to change. The learning goals below should be viewed as the key concepts you should grasp after each week, and also as a study guide before each exam, and at the end of the semester. Each exam will test on the material that was taught up until the review day prior to the exam. The applications in the second half of the semester tend to build on the concepts in the first half of the semester though, so it is still important to review those concepts throughout the semester.

Week 01, 01/14 - 01/18: § **6.1 Inverse Functions**; § **6.2 Exponential Functions**; § **6.3 Review of Logarithms**

- Recognize where a function has an inverse and be able to compute it, if possible.
- State and apply the Inverse Function Theorem. Give an example which illustrates it graphically.
- Graph exponential and logarithmic functions and explain how $\log_b(x)$ is the inverse of b^x for $b > 0,\ b \neq 1$.
- Compute the derivatives/integrals of exponential functions and apply the chain rule to functions involving compositions of these functions.
- Solve logarithmic and exponential equations.

Week 02, 01/21 - 01/25: **NO CLASS MONDAY**; § **6.4 Calculus with Logarithms**; § **6.6 InverseTrigonometric Functions**; Friday is the last day to change grade mode on LoboWEB

- Compute the derivatives logarithmic functions and apply the chain rule to functions involving compositions of these functions.
- Compute integrals of the form $\int \frac{du}{u}$ or use substitution to put integrals into this form.
- Graph $\arcsin x,\ \arccos x,\ \arctan x$ and explain where their domain/range restrictions come from.
- Evaluate inverse trig functions at standard values (such as $\arcsin(-0.5)$).
- Compute the derivatives of the inverse trigonometric functions. Apply the chain rule to functions involving compositions of one or more of these functions.
- Compute integrals of functions which have antiderivatives involving inverse trigonometric functions (for example $\int \frac{e^{3x}}{e^{6x}+1}\,dx$).

Week 03, 01/28 - 02/01: § **6.7 Hyperbolic Functions**; § **6.8 L'Hospital's Rule**; **Review for Exam 1**; Friday is the last day to drop without a "W" grade.

- Apply L'Hospital's when applicable. If possible, rewrite functions into a form so that the rule can be applied.
- Know the definitions of $\sinh x$, $\cosh x$ and $\tanh x$ in terms of exponential functions and use these to compute their derivatives as well as to determine their long term behavior.
- Apply L'Hospital's when applicable. If possible, rewrite functions into a form so that the rule can be applied.
- Compare growth rates of various functions using infinite limits.

Week 04, 02/04 - 02/08: **Exam 1 (Monday)**; § **7.1 Integration by Parts**; § **7.2 Trigonometric Integrals**

- Integrate functions using the method of integration by parts (be able to derive the formula from the product rule!).
- Use appropriate trigonometric identities and substitutions to evaluate trigonometric integrals.

Week 05, 02/11 - 02/15: § **7.3 Trigonometric Substitution**; § **7.4 Partial Fraction Decomposition**

- Integrate functions using an appropriate trigonometric substitution: $a^2 - x^2$ vs $a^2 + x^2$ vs $x^2 - a^2$.
- Integrate functions using a partial fraction decomposition (there are four cases, you should know them well!).

Week 06, 02/18 - 02/22: § **7.7 Numerical Integration**; § **7.8 Improper Integrals**

- Derive and use the Trapezoid rule to approximate the value of a definite integral. Compare this to the rectangle approximation rules. If time permits, discuss the error in this approximation and compare it to the error used in the Midpoint rule.
- Recognize an improper integral (infinite limits of integration and/or discontinuities in the domain of the integrand) and determine whether it diverges or converges. Determine the exact value if possible.

Week 07, 02/25 - 03/01: § **9.1 Intro to Differential Equations**; § **9.2 Direction Fields**; § **9.3 Separation of Variables**

- Define what a differential equation is and be able to give and solve examples of first and second order intial value problems (masses on springs, Newton's Law of Cooling, etc...).
- Plot a direction field by hand, using Matlab or Desmos, or some other graphing utility. Use this to describe the behavior of solutions.
- Identify when a differential equation is separable and use the separation of variables method to solve the equation or initial value problem.

Week 08, 03/04 - 03/08: § **9.4 Logistic Growth**; **Review for Exam 2**; **Exam 2 (Friday)**

- Describe how the exponential growth model differs from the logistic growth model. Be able to solve both types of differential equations and describe the long term behavior of the solutions.
- Determine the carrying capacity of a population modeled by the logistic equation (be able to do it by solving the equation explicitly and also by using a direction field).

Week 09, 03/11 - 03/15: **SPRING BREAK**

Week 10, 03/18 - 03/22: § **11.1 Sequences**; § **11.2 Intro to Series**

- Use limits to determine whether a sequence of real numbers converges or diverges.
- Use limits to determine whether a sequence of real numbers is increasing/decreasing/bounded.
- Explain what it means for a sequence to be monotonic and give two examples: the first being a monotonically increasing sequence which diverges, and the other a sequence which increases monotonically and converges. Be able to prove a sequence is monotonic!
- Find a formula or recurrence relation to define all the terms of a given sequence.
- Explain how the convergence or divergence of an infinite series $\sum_{k=1}^{\infty} a_n$ is defined in terms of the sequence of its partial sums.
- Define a geometric series. Under what conditions do geometric series converge? Diverge? How do you find the sum of a convergent geometric series? Give several examples.
- Define a telescoping series. Describe how to determine whether or not it converges. Can you compute the exact value of a convergent telescoping series? Explain.

Week 11, 03/25 - 03/29: § **11.3 Integral Test**; § **11.4 Comparison Tests** ;§ **11.5 Alternating Series**

- Recognize when the Integral Test can be applied to a series and use it to determine whether or not a series converges or diverges. Does the integral test give you the exact value of a convergent series? Explain.
- Define the *Harmonic Series* and use the Integral Test to determine whether it converges of diverges.
- Recognize p-series and know when they converge or diverge.
- Describe the difference between the Direct and Limit Comparison tests. When might one be better to use than the other?
- Recognize when the Comparison Tests can be applied to a series and use it to determine whether or not a series converges or diverges. Do the Comparison Tests give you the exact value of a convergent series? Explain.
- Recognize when the Alternating Series Test can be applied to a series and use it to determine whether the series converges or diverges.
- State the Alternating Series Estimation Theorem and use it to approximate the sum of a series.

Week 12, 04/01 - 04/05: § **11.6 Absolute Convergence/Ratio Test**; § **11.7 Strategies**§ **11.8 Power Series**

- Describe the difference between what means for a series to be *Conditionally Convergent* vs *Absolutely Convergent*. Be able to give several examples to illustrate this.
- Recognize when the Ratio Test can be applied to a series and use it to determine whether the series converges or diverges.
- Given an infinite series, be able to determine: 1. Which tests may be used to determine whether it converges or diverges; 2. It is possible to find the exact value of the series if it converges?; 3. If it converges, does it converge absolutely or conditionally?
- Recognize Power Series and be able to determine the interval and radius of convergence for a given series. (What are the three possibilities?)

Week 13, 04/08 - 04/12: § **11.9 Functions as Power Series**; § **11.10 Taylor Series**; Friday is the last day to drop without the Dean's permission.

- Know when a power series can be integrated/differentiated term-by-term. What happens to the radius of convergence? What about the interval of convergence? What about the index of summation?
- Represent a functions as a power series and determine the interval of convergence (for example things like $\frac{x^3}{1+x^2}$, $\tan^{-1} x$, $\frac{3}{(1+x)^2}$).
- Define the *Taylor Series of* $f(x)$ *centered at* $x = a$ (the case $a = 0$ is called a *Maclaurin Series*).
- Be able to derive the general formula for the nth coefficient of a Taylor Series (the derivation begins on page 777 in the text).
- Compute the nth order Taylor Polynomial $T_n(x)$ at $x = a$.
- Find the Taylor series for a function $f(x)$ at a specified value a (for example $f(x) = 1/x$, $a = -3$). Determine the interval/radius of convergence.
- Know the Maclaurin series for e^x, $\cos x$, $\sin x$, $\frac{1}{1-x}$ and use them to construct power series for functions that are combinations of these.
- Use power series to evaluate limits and estimate the value of integrals when possible.

Week 14, 04/15 - 04/19: § **11.11 Error Estimates with Taylor Approximations**; **Review for Exam 3**; **Exam 3 (Friday)**

- State and use *Taylor's Theorem/Inequality* (page 780) to determine the error in approximating $f(x)$ by its Taylor polynomial $T_n(x)$ at $x = a$.

Week 15, 04/22 - 04/26: § **11.11 Applications of Series**; **Complex Numbers**

- Apply series to solve physical application problems.
- Use series to derive Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$.
- Perform algebraic operations on ccomplex numbers (addition/subtraction, multiplication/division, roots/powers)

Week 16, 04/29 - 05/03: **Review Week**; Friday is the last day to change grade mode with form and last day to drop with Dean's permission.

Week 17, 05/06 - 05/10: **Final Exam on Monday 5/6 at 10:00 AM, Room TBA**

Weekly Homework

This schedule is tentative and subject to change. The "Hand in" problems are to be turned in for a grade (Week 01 problems are turned in on Monday of Week 02 class, etc.). These problems are graded so that you can receive feedback regarding your understanding of concepts and use of notation. The "Self-Check" problems are odd problems in the book that test your basic knowledge of the material. It is imperative that you do **ALL** of the Self-Check problems as well. It may be tempting to skip the Self-Check Problems because they are not turned in. However, the primary goal of this course is to learn to solve problems and demonstrate that knowledge on exams, and the best way to accomplish this goal is by understanding all of the homework. The collected problems alone are not intended to give you enough practice to learn calculus, so if you ignore the Self-Check Problems, you will make the course far more difficult for yourself. The recitation quizzes, midterms, and final exam will be based on both sets of problems.

Week 01, 01/14 - 01/18: § **6.1 Inverse Functions**; § **6.2 Exponential Functions**; § **6.3 Review of Logarithms**

- § **6.1**: Self-Check:1-27, **Hand in**: 20, 24, 26, 36, 38, 40
- § **6.2**: Self-Check:1-49, **Hand in**: 26, 28, 32, 36, 38, 46, 50,54, 58, 64, 70, 82, 84, 90
- § **6.3**: Self-Check: 1-35, **Hand in**: 28, 36, 40, 46, 56, 64

Week 02, 01/21 - 01/25: **NO CLASS MONDAY**; § **6.4 Calculus with Logarithms**; § **6.6 InverseTrigonometric Functions**; Friday is the last day to change grade mode on LoboWEB

- § **6.4**: Self-Check: 1-15, 71-75, **Hand in**: 10, 12, 26, 38, 50, 52, 62, 72, 78, 80
- § **6.6**: Self-Check: 1-15, **Hand in**: 12, 22, 34, 38, 46, 48, 60, 62, 68, 70

Week 03, 01/28 - 02/01: § **6.7 Hyperbolic Functions**;§ **6.8 L'Hospital's Rule**; **Review for Exam 1**; Friday is the last day to drop without a "W" grade.

- § **6.7**: Self-Check: 1-13, **Hand in**: 30, 32, 38, 49, 54
- § **6.8**: Self-Check: 1-25, **Hand in**: 12, 14, 22, 28, 40, 42, 44, 46, 48, 50, 58, 62, 86, 92

Week 04, 02/04 - 02/08: **Exam 1 (Monday)**; § **7.1 Integration by Parts**; § **7.2 Trigonometric Integrals**

- § **7.1**: Self-Check: 1-23, **Hand in**: 10, 12, 18, 20, 26, 34, 36, 42
- § **7.2**: Self-Check: 1-11, **Hand in**: 4, 6, 12, 20, 22, 34, 62, Compute $\int \sec x \, dx$

Week 05, 02/11 - 02/15: § **7.3 Trigonometric Substitution**; § **7.4 Partial Fraction Decomposition**

- § **7.3**: Self-Check: 1-19, **Hand in**: 6, 12, 14, 16
- § **7.4**: Self-Check: 1-17, **Hand in**: 10, 12, 14, 16, 22, 24, 30, 34, 54

Week 06, 02/18 - 02/22: § **7.7 Numerical Integration**; § **7.8 Improper Integrals**

- § **7.7**: Self-Check: 1-3, **Hand in**: 10(a,b), 16(a,b), 31, 44
- § **7.8**: Self-Check: 1-13, **Hand in**: 8, 10, 14, 16, 18, 22, 24, 26, 28, 43, 40

Week 07, 02/25 - 03/01: § **9.1 Intro to Differential Equations**; § **9.2 Direction Fields**; § **9.3 Separation of Variables**

- § **9.1**: Self-Check: 1-7, **Hand in**: 4, 8, 9, 10, 12, 14
- § **9.2**: Self-Check: 1-5, **Hand in**: 2-6 (all)
- § **9.3**: Self-Check: 1-13, **Hand in**: 10, 12, 16, 18, 39, 48

Week 08, 03/04 - 03/08: § **9.4 Logistic Growth**; **Review for Exam 2**; **Exam 2 (Friday)**

- § **9.4**: Self-Check: 1-3, **Hand in**: 4, 10, 20(a,b)

Week 09, 03/11 - 03/15: **SPRING BREAK**

Week 10, 03/18 - 03/22: § **11.1 Sequences**; § **11.2 Intro to Series**

- § **11.1**: Self-Check: 1-55, **Hand in**: 2, 6, 14, 18, 26, 34, 36, 42, 46, 64, 67, 76, 78
- § **11.2**: Self-Check: 1-31, **Hand in**: 1, 2, 8, 15, 18, 20, 28, 32, 36, 38, 40, 41, 52, 58

Week 11, 03/25 - 03/29: § **11.3 Integral Test**; § **11.4 Comparison Tests** ;§ **11.5 Alternating Series**

- § **11.3**: Self-Check: 1-25, **Hand in**: 6, 12, 16, 24, 28
- § **11.4**: Self-Check: 1-31, **Hand in**: 2, 4, 8, 10, 14, 18, 24, 28, 31, 32
- § **11.5**: Self-Check: 1-27, **Hand in**: 2, 8, 14, 18, 24, 28

Week 12, 04/01 - 04/05: § **11.6 Absolute Convergence/Ratio Test**; § **11.7 Strategies**§ **11.8 Power Series**

- § **11.6**: Self-Check: 1-25, **Hand in**: 8, 10, 12, 18, 24, 35, 36, 42
- § **11.7**: Self-Check: 1-37 **Hand in**: 4, 6, 8, 10, 26, 38
- § **11.8**: Self-Check: 1-25, **Hand in**: 6, 10, 16, 20, 30

Week 13, 04/08 - 04/12: § **11.9 Functions as Power Series**; § **11.10 Taylor Series**; Friday is the last day to drop without the Dean's permission.

- § **11.9**: Self-Check: 1-15, **Hand in**: 6, 8, 13, 14, 16, 26, 30
- § **11.10**: Self-Check: 1-19, **Hand in**: 2, 4, 8, 16, 20, 30, 32, 48, 54, 56, 64, 66

Week 14, 04/15 - 04/19: § **11.11 Error Estimates with Taylor Approximations**; **Review for Exam 3**; **Exam 3 (Friday)**

- § **11.11**: Self-Check: 1-9, **Hand in**: 4, 8, 10, 14, 16

Week 15, 04/22 - 04/26: § **11.11 Applications of Series**; **Complex Numbers**

- § **11.11**: Self-Check: 1-9, **Hand in**: 26, 28, 32, 33
- § **Appendix H**: Self-Check: 1-45, **Hand in**: 2, 8, 16, 22, 24, 26, 28, 32, 34, 43, 44, 48