University of New Mexico-Math 264: Calculus III

Spring 2019

Course Description

Vector operations, vector representation of planes and curves, functions of several variables, partial derivatives, gradient, tangent planes, optimization, multiple integrals in Cartesian, cylindrical and spherical coordinates, vector fields, line integrals and Green's theorem.

Required Materials

- *Calculus*, Stewart, 7e

Prerequisites

Prerequisites: 163 with a grade of "C" (not "C-") or better.

Course Objectives

1. (**Vector Operations**) Perform basic operations on vectors in 3D: addition, subtraction, scalar multiplication, dot product. Visualize addition, subtraction and scalar multiplication geometrically, state geometric meaning of dot product and crossproduct, recognize and write down the equations defining lines and planes, and draw geometric information from the equations (such as a point on lines/planes, tangent and normal vectors, intersections)

2. (**Vector-valued Functions of One Variable**) Visualize given functions as curves in space, find functional parametrization of given curves, find their derivatives and interpret them as tangent vectors to curves; for functions describing the motion of a particle, interpret derivatives as velocity and acceleration; solve initial value problems.

3. (**Scalar-valued Functions of Several Variables**) Visualize functions of two variables by graphs in space or level curves in the plane; visualize functions of three variables by level

surfaces in space; recognize and graph equations for conic sections and for surfaces of revolution; state what it means for a limit of a function of several variables to exist; compute partial derivatives, gradients, directional derivatives and understand their meanings, e.g. with respect to direction of fastest growth and tangent planes; compute the gradient of a function and state its geometric significance; solve min/max problems with or without constraints (using substitution or Lagrange multipliers for the former) explain why the Lagrange multiplier method works.

4. (**Double and Triple Integrals**) Compute by reducing to an iterated integral, by changing the order of integration, by changing from Cartesian coordinates to cylindrical or spherical coordinates and vice-versa; use double and triple integrals to compute areas, volumes, centers of mass.

5. (**Vector Fields**) Visualize basic vector fields by flow lines and integral curves; state the definition of a gradient (or conservative) vector field and how to recognize one and compute a potential function; compute the divergence and curl of a vector field; rules for differentiation; recognize permissible and nonpermissible operations.

6. (**Line Integrals**) Compute line integrals such as arclength, work, circulation using the parametrization of a curve; compute using the Fundamental Theorem of Line Integrals when applicable; state Green's theorem (2-D), apply it to examples.

Grading Policy

You grade will be based on the following:

- **3 in-class exams** (100 points each). Exams will be based on homework and examples discussed in class. No calculators, notes, or any kind of electronic device may be used on exams. There are **NO MAKEUP EXAMS UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**.

- **Written Homework** (50 points) HW is due at the beginning of class. e-mailed HW will not be accepted. Your written work should be neatly organized, showing all steps and using proper mathematical notation. Your papers must be stapled. In order to ensure that assignments are graded promptly, and to discourage students from falling behind, **LATE ASSIGNMENTS WILL NOT BE ACCEPTED UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**. However, in recognition of the fact that unavoidable issues sometimes arise, the lowest of each student's homework grades (including zeros for unsubmitted assignments) will be dropped when calculating final semester grades. Despite this policy, you should complete every assignment, even if you miss a deadline, because understanding the homework will help you perform well on exams.

- **Recitation** (100 points) Students will receive a grade for each recitation that will be based partially on attendance, and partially on quizzes, participation, group problem-solving, or any combination of these. See the Recitation Handout for more infomation. There will be 12 quizzes with the lowest score dropped. There are **NO MAKEUP QUIZZES UNDER**

ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS.

- **Final Exam** (200 points) The final exam will be cumulative and held on Monday, May 6th at 7:30 AM. **NO EARLY FINAL EXAMS NOR MAKE UP EXAMS WILL BE GIVEN UNDER ANY CIRCUMSTANCES OUTSIDE OF A UNIVERSITY-EXCUSED ABSENCE OR A VERIFIABLE DOCUMENTED EMERGENCY OR ILLNESS**.

Course Policies

Student Behavior

All students must abide by the Student Code of Conduct: pathfinder.unm.edu. According to the Code of Conduct, student activities that interfere with the rights of others to pursue their education or to conduct their University duties and responsibilities will lead to disciplinary action. This includes any activities that are disruptive to the class and any acts of academic dishonesty. Students are expected to behave in a courteous and respectful manner toward the instructor and their fellow students. The use of cell phones, headphones, smart watches, etc. is not permitted during class or exams. .

Attendance Policy

Attendance is mandatory. If a student has four or more unexcused absences he/she may be dropped from the course. Tardiness or early departure may be regarded as an absence. After the Withdrawal deadline the instructor will not drop any student. Please note that it is the student's responsibility to drop the course if he/she stops attending. A failing grade of F may be assigned if the student stops attending and does not drop before the posted deadline. **No early final exams** will be given except in documented emergencies: flight reservations, weddings, vacations, birthdays, non-NCAA sporting events etc. are not considered emergencies.

Policies on Grading Options

You must select your grade mode (Letter Grade, CR/NC, or Audit) within the first 2 weeks of the semester. We will not give permission to change the grade mode after the deadline. Students who are in the regular grade mode and who withdraw after the end of week 3 will receive a grade of W. If you do not withdraw, you will receive a letter grade of A, B, C, D, or F (not a W). Students who are in the CR/NC grade mode and who withdraw after the end of week 3 will receive a grade of W. If you do not withdraw, you will receive a letter grade of NC (not a W). See the list of all deadlines: registrar.unm.edu.

Deadlines

The Department of Mathematics and Statistics will adhere to all of the registration deadlines published by the Office of the Registrar in the schedule of classes: registrar.unm.edu . We will not give permission to override any deadline except in documented emergencies.

Academic Integrity and Honesty

Cheating of any kind will not be tolerated. Examples are: looking at a neighbor's exam, plagiarizing, using a calculator when not permitted, using the book and/or a cheat sheet, modifying an exam after it is graded, etc. The instructor may warn an offending student, the score of the exam may be reduced, the score may be set to zero, the student may be dropped from the class, the student may get a grade of F for the class, and in most cases the incident will be reported to the Dean of Students..

Accommodations for Disabilities

We will accommodate students with documented disabilities through the Accessibility Resource Center (ARC). During the first two weeks of the semester, those students should inform the instructor of their particular needs.

Title IX

In an effort to meet obligations under Title IX, UNM faculty, Teaching Assistants, and Graduate Assistants are considered "responsible employees" by the Department of Education. This designation requires that any report of gender discrimination which includes sexual harassment, sexual misconduct and sexual violence made to a faculty member, TA, or GA must be reported to the Title IX Coordinator at the Office of Equal Opportunity, oeo.unm.edu. For more information on the university policy regarding sexual misconduct, see: https://policy.unm.edu/university-policies/2000/2740.html.

Schedule and Weekly Learning Goals

This schedule is tentative and subject to change. The learning goals below should be viewed as the key concepts you should grasp after each week, and also as a study guide before each exam, and at the end of the semester. Each exam will test on the material that was taught up until the review day prior to the exam. The applications in the second half of the semester tend to build on the concepts in the first half of the semester though, so it is still important to review those concepts throughout the semester.

Week 01, 01/14 - 01/18: § **12.1 3-D Space:** $\mathbb{R}^3$; § **12.2 Vectors**; § **12.3 Dot Product**

- Write vectors in component form and as linear combinations of the standard unit vectors.
- Compute the magnitude and direction of vectors.
- Compute sums and differences of vectors algebraically and geometrically.
- Compute dot products, determine angles between vectors. Know the geometric interpretation of the dot product.
- Compute vector projections.

Week 02, 01/21 - 01/25: **NO CLASS MONDAY**; § **12.4 Cross Product**; § **12.5 Lines**; Friday is the last day to change grade mode on LoboWEB.

- Compute cross products. Know the geometric interpretation of the cross product.
- Write parametric and symmetric equations of a line in $\mathbb{R}^3$.
- Write parametric equations for a line segment with initial point $A(x_1, y_1, z_1)$ and terminal point $B(x_2, y_2, z_2)$.

Week 03, 01/28 - 02/01: § **12.5 Planes** ; § **12.6 Quadric Surfaces**; **Review for Exam 1**; Friday is the last day to drop without a "W" grade.

- Write the equation of a plane in $\mathbb{R}^3$.
- Determine distances between objects in $\mathbb{R}^3$ (lines, planes, points, spheres).
- determine equations for the intersection of objects in $\mathbb{R}^3$.
- Identify quadric surfaces and sketch graphs in $\mathbb{R}^3$ by using traces.
- Identify and sketch cylinders in $\mathbb{R}^3$.

Week 04, 02/04 - 02/08: **Exam 1 (Monday)**; § **13.1 Vector Functions**; § **13.2 Calculus with Vector Functions**

- Find the domains of vector functions. Sketch graphs of these functions in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- Compute limits, derivatives, and integrals of vector functions.
- Compute the unit tangent vector to a curve at a given point.

Week 05, 02/11 - 02/15: § **13.3 Arc Length/Curvature**; § **13.4 Motion in Space**

- Compute the arc length and curvature of a curve.
- Parametrize a curve with respect to arc length.
- Compute the unit tangent, normal, and binormal vectors to a curve at a point.
- Find the tangential and normal components of acceleration.
- Solve projectile motion problems in $\mathbb{R}^2$.

Week 06, 02/18 - 02/22: §§ **14.1/14.2 Multivariable Functions/Limits**; § **14.3 Partial Derivatives**

- Determine the domains and ranges of multivariable functions. Sketch level curves and surfaces of multivariable functions.
- Use limits to determine whether a multivariable function is continuous at a point.
- Be able to show when a limit of a multivariable function does not exist.
- Compute partial derivatives of multivariable functions (including implicit differentiation as well as higher order partial derivatives).
- Determine when a multivariable function is differentiable.
- Estimate the value of partial derivatives at a point, using a table or a graph.
- Be able to apply and interpret partial derivatives in application problems.

Week 07, 02/25 - 03/01: § **14.4 Linearization**; § **14.5 Chain Rule**; § **14.6 Directional Derivatives**

- Compute the linear approximation (linearization) and differential of a multivariable function. Understand where the differential comes from in terms of its linear approximation.
- Interpret the linear approximation of a function of two variables as the plane tangent to a surface at a given point.
- Use linear approximations/differentials to estimate the maximum error in the calculated value of a physical quantity (like area or volume, for example).
- Draw and use a tree diagram to write out an appropriate version of the chain rule for multivariable functions.
- Use the chain rule to derive the *Implicit Function Theorem*. Apply this to functions of two and three variables.
- Compute the directional derivative and the gradient of multivariable functions. Be able to estimate the value of the directional derivative at a point using a table or a graph.

Week 08, 03/04 - 03/08: § **14.6 Directional Derivatives**; **Review for Exam 2**; **Exam 2 (Friday)**

- Use the gradient to determine in what direction(s) a function increases/decreases most rapidly at a point.
- Use the gradient to compute the normal line and tangent plane to a surface at a point. Be sure to understand how this relates to the equation of the tangent plane used in § 14.4.

Week 09, 03/11 - 03/15: **SPRING BREAK**

Week 10, 03/18 - 03/22: § **14.7 Extrema/Saddle Points** ; § **14.8 Lagrange Multipliers**

- Determine critical points of a function of two variables and classify them as local maxima, minima, or saddle points.
- Determine the absolute extrema of a function of two variables on a closed and bounded domain.
- Use the method of Lagrange Multipliers (one or two constraints) to solve optimization problems. You should understand why this works!!!!

Week 11, 03/25 - 03/29: §§ **15.1/15.2 Intro to Double Integrals (over rectangles, iterated integrals)**; § **15.3 Double Integrals (General Regions)**

- Define double integrals over rectangles in $\mathbb{R}^2$.
- Estimate the value of a double integral using the midpoint method.
- Evaluate double integrals over rectangles using iterated integrals in $\mathbb{R}^2$.
- State and apply *Fubini's Theorem* to double integrals over rectangles.
- Describe a region in $\mathbb{R}^2$ as a Type I or Type II region using inequallities. Be able to go back and forth between these if possible.
- Evaluate double integrals over Type I or Type II regions. Be able to reverse the order of integration when necessary or helpful (or just for practice).
- Evaluate $\int\int_R f(x,y)\,dA$ and determine if it is a volume in $\mathbb{R}^3$. Provide a sketch of the region in $\mathbb{R}^3$ for familiar objects like spheres, cones, paraboloids etc.

Week 12, 04/01 - 04/05: § **15.4 Double Integrals (Polar Coordinates)**; § **15.7 Triple Integrals** § **15.8 Triple Integrals (Cylindrical Coordinates)**

- Describe a region in $\mathbb{R}^2$ using polar coordinate inequalities.
- Evaluate a double integral over a general region by converting it to polar coordinates.
- Find the area of a region in $\mathbb{R}^2$ using a double integral.
- Define and evaluate triple integrals in rectangular coordinates. Be able to integrate in various orders, e.g. $dz\,dy\,dx$ or $dz\,dx\,dy$ or $dx\,dz\,dy$ by drawing a sketch of the region of integration.
- Compute the volume of a region D in $\mathbb{R}^3$ as $\int\int\int_D 1\,dV$

Week 13, 04/08 - 04/12: § **15.9 Triple Integrals (Spherical Coordinates)**; **Review for Exam 3**; **Exam 3 (Friday)**; Friday is the last day to drop without the Dean's permission.

- Evaluate triple integrals by converting to spherical coordinates.
- Given $\int\int\int_B f(x,y,z)\,dV$, be able to convert it to any of the three coordinate systems.

Week 14, 04/15 - 04/19: § **16.1 Vector Fields**; § **16.5 Curl/Divergence**; § **16.2 Line Integrals**

- Sketch vector fields in $\mathbb{R}^2$ and $\mathbb{R}^3$ (for $\mathbb{R}^3$ sketch the gravitational field).
- Compute gradient fields and define what a conservative vector field is.
- Compute the curl and divergence of a vector field. What is curl($\nabla\mathbf{F}$)? div(curl $\mathbf{F}$)?
- Evaluate line integrals of scalar functions with respect to arc length: $\int_C f(x,y)\,ds$ and $\int_C f(x,y,z)\,ds$
- Evaluate line integrals with respect to a variable: e.g. $\int_C P(x,y)\,dx$ or $\int_C P(x,y)\,dx + Q(x,y)\,dy$

Week 15, 04/22 - 04/26: § **16.2 Line Integrals**; § **16.3 Fundamental Theorem of Line Integrals**; § **16.4 Green's Theorem**

- Evaluate line integrals of vector fields $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C P\,dx + Q\,dy + R\,dz = \int_C \mathbf{F}\cdot\hat{\mathbf{T}}\,ds$
- Use a line integral to compute the work done by a field on a particle moving along a piecewise smooth curve C.
- Use a sketch of a vector field to determine whether a line integral of a vector field is positive, negative, or zero.
- Compute potential functions of simple conservative vector fields by inspection.
- Determine if a vector field is conservative and find a potential function.
- State the *Fundamental Theorem for Line Integrals* and apply it appropriately. Describe some physical applications of this (like conservation of energy).
- State *Green's Theorem* and apply it appropriately.

Week 16, 04/29 - 05/03: **Review Week**; Friday is the last day to change grade mode with form and last day to drop with Dean's permission.

Week 17, 05/06 - 05/10: **Final Exam on Monday 5/6 at 7:30 AM, Room TBA**

Weekly Homework

This schedule is tentative and subject to change. The "Hand in" problems are to be turned in for a grade (Week 01 problems are turned in on Monday of Week 02 class, etc.). These problems are graded so that you can receive feedback regarding your understanding of concepts and use of notation. The "Self-Check" problems are odd problems in the book that test your basic knowledge of the material. It is imperative that you do **ALL** of the Self-Check problems as well. It may be tempting to skip the Self-Check Problems because they are not turned in. However, the primary goal of this course is to learn to solve problems and demonstrate that knowledge on exams, and the best way to accomplish this goal is by understanding all of the homework. The collected problems alone are not intended to give you enough practice to learn calculus, so if you ignore the Self-Check Problems, you will make the course far more difficult for yourself. The recitation quizzes, midterms, and final exam will be based on both sets of problems.

Week 01, 01/14 - 01/18: § **12.1 3-D Space:** $\mathbb{R}^3$; § **12.2 Vectors**; § **12.3 Dot Product**

- § **12.1**: Self-Check: 1-33, **Hand in**: 6, 8, 10, 14, 18, 20, 41
- § **12.2**: Self-Check: 1-31, **Hand in**: 6, 8, 22, 24, 34, 42, 44
- § **12.3**: Self-Check: 1-27 , **Hand in**: 1, 10, 12, 14, 20, 24, 26, 44, 50, 62, 64

Week 02, 01/21 - 01/25: **NO CLASS MONDAY**; § **12.4 Cross Product**; § **12.5 Lines**; Friday is the last day to change grade mode on LoboWEB.

- § **12.4**: Self-Check: 1-19, **Hand in**: 6, 14, 18, 20, 32, 45
- § **12.5**: Self-Check:1–21, **Hand in**: 4, 6, 10, 12, 18, 20, 48, 64, 70

Week 03, 01/28 - 02/01: § **12.5 Planes** ; § **12.6 Quadric Surfaces**; **Review for Exam 1**; Friday is the last day to drop without a "W" grade.

- § **12.5**: Self-Check: 23-41, **Hand in**: 24, 26, 28, 32, 34, 54, 74
- § **12.6**: Self-Check: 1-27, **Hand in**: 4, 10, 11, 16, 18, 21-28 (all), 42

Week 04, 02/04 - 02/08: **Exam 1 (Monday)**; § **13.1 Vector Functions**; § **13.2 Calculus with Vector Functions**

- § **13.1**: Self-Check: 1-19, **Hand in**: 4, 6, 10, 16, 21-26 (all), 30, 42, 47, 48
- § **13.2**: Self-Check: 1-17, 35-39, **Hand in**: 4, 6, 10, 20, 26, 34, 36, 38, 40

Week 05, 02/11 - 02/15: § **13.3 Arc Length/Curvature**; § **13.4 Motion in Space**

- § **13.3**: Self-Check: 1-21, 31, **Hand in**: 4, 15, 16, 18, 22, 24, 30, 42, 48
- § **13.4**: Self-Check: 1-15, **Hand in**: 2, 8, 16, 22, 26, 38

Week 06, 02/18 - 02/22: §§ **14.1/14.2 Multivariable Functions/Limits**; § **14.3 Partial Derivatives**

- § **14.1**: Self-Check: 1-31, **Hand in**: 2, 4, 14, 22, 24, 32, 34, 36, 46
- § **14.2**: Self-Check: 1-15, **Hand in**: 6, 8, 14, 18, 30, 34
- § **14.3**: Self-Check: 1-53, **Hand in**: 4, 6, 10, 22, 24, 30, 42, 50, 52, 66, 74, 82, 87

Week 07, 02/25 - 03/01: § **14.4 Linearization**; § **14.5 Chain Rule**; § **14.6 Directional Derivatives**

- § **14.4**: Self-Check: 1-23, **Hand in**: 4, 14, 18, 21, 22, 33, 34, 39
- § **14.5**: Self-Check: 1-13, 17-25, **Hand in**: 2, 10, 18, 22, 28, 34, 36, 40
- § **14.6**: Self-Check: 1-29, **Hand in**: 2, 4, 8, 14, 22, 30, 32

Week 08, 03/04 - 03/08: § **14.6 Directional Derivatives**; **Review for Exam 2**; **Exam 2 (Friday)**

- § **14.6**: Self-Check: 1-29, **Hand in**: 33, 34, 38, 44, 46, 60

Week 09, 03/11 - 03/15: **SPRING BREAK**

Week 10, 03/18 - 03/22: § **14.7 Extrema/Saddle Points** ; § **14.8 Lagrange Multipliers**

- § **14.7**: Self-Check: 1-17, 29-35, **Hand in**: 8, 10, 12, 16, 18, 30, 34, 40
- § **14.8**: Self-Check: 1-19, **Hand in**: 1, 4, 6, 8, 10, 16, 20

Week 11, 03/25 - 03/29: §§ **15.1/15.2 Intro to Double Integrals (over rectangles, iterated integrals)**; § **15.3 Double Integrals (General Regions)**

- § **15.1**: Self-Check: 1-11, **Hand in**: 2, 6, 8, 12
- § **15.2**: Self-Check: 1-25, **Hand in**: 10, 12, 20, 24, 30, 36
- § **15.3**: Self-Check: 1-31, 37, 43-53, **Hand in**: 8, 16, 20, 22, 26, 38, 48, 52, 54

Week 12, 04/01 - 04/05: § **15.4 Double Integrals (Polar Coordinates)**; § **15.7 Triple Integrals**; § **15.8 Triple Integrals (Cylindrical Coordinates)**

- § **15.4**: Self-Check: 1-13, 19-31, **Hand in**: 8, 10, 22, 26, 30, 32
- § **15.7**: Self-Check: 1-21, 27-35, **Hand in**: 12, 14, 18, 22, 28, 34, 54
- § **15.8**: Self-Check: 1-29, **Hand in**: 4, 10, 12, 18, 22, 24, 30

Week 13, 04/08 - 04/12: § **15.9 Triple Integrals (Spherical Coordinates)**; **Review for Exam 3**; **Exam 3 (Friday)**; Friday is the last day to drop without the Dean's permission.

- § **15.9**: Self-Check: 1-27, 35, 39, 41, **Hand in**: 4, 6, 10, 12, 16, 18, 20, 22, 30, 36, 40

Week 14, 04/15 - 04/19: § **16.1 Vector Fields**; § **16.5 Curl/Divergence**; § **16.2 Line Integrals**

- § **16.1**: Self-Check: 1-9, 11-18 (all), 21-25, **Hand in**: 6, 8, 11-18 (all), 24, 26, 29-32 (all)
- § **16.5**: Self-Check: 1-19, **Hand in**: 2, 6, 9-12 (all), 16, 18, 20, 30
- § **16.2**: Self-Check: 1-21, **Hand in**: 4, 6, 8, 12, 14

Week 15, 04/22 - 04/26: § **16.2 Line Integrals**; § **16.3 Fundamental Theorem of Line Integrals**; § **16.4 Green's Theorem**

- § **16.2**: Self-Check: 1-21, **Hand in**: 16, 18, 20, 22, 42, 44, 45, 46
- § **16.3**: Self-Check: 1-25, **Hand in**: 2, 4, 6, 8, 14, 18, 20, 22, 25, 26
- § **16.4**: Self-Check: 1-13, **Hand in**: 2, 6, 8, 12, 14, 18