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This is Exercise 1.12.2 in the notes. These are things you should have learned in the prerequisite course.

The information below relates $y$, a second measurement on wood volume, to $x_{1}$, a first measurement on wood volume, $x_{2}$, the number of trees, $x_{3}$, the average age of trees, and $x_{4}$, the average volume per tree. Note that $x_{4}=x_{1} / x_{2}$. Some of the information has not been reported, so that you can figure it out on your own.

Table of Coefficients

| Predictor | $\hat{\beta}_{k}$ | $\mathrm{SE}\left(\hat{\beta}_{k}\right)$ | $t$ | $P$ |
| :--- | ---: | ---: | :---: | :---: |
| Constant | 23.45 | 14.90 |  | 0.122 |
| $x_{1}$ | 0.93209 | 0.08602 |  | 0.000 |
| $x_{2}$ |  | 0.4721 | 1.5554 | 0.126 |
| $x_{3}$ | -0.4982 | 0.1520 |  | 0.002 |
| $x_{4}$ | 3.486 | 2.274 |  | 0.132 |

Analysis of Variance

| Source | $d f$ | $S S$ | $M S$ | $F$ | $P$ |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Regression | 4 | 887994 |  |  | 0.000 |
| Error |  |  |  |  |  |
| Total | 54 | 902773 |  |  |  |


|  | Sequential |  |
| :--- | ---: | ---: |
| Source | $d f$ | $S S$ |
| $x_{1}$ | 1 | 883880 |
| $x_{2}$ | 1 | 183 |
| $x_{3}$ | 1 | 3237 |
| $x_{4}$ | 1 | 694 |

(a) How many observations are in the data?
(b) What is $R^{2}$ for this model?
(c) What is the mean squared error?
(d) Give a $95 \%$ confidence interval for $\beta_{2}$.
(e) Test the null hypothesis $\beta_{3}=0$ with $\alpha=0.05$.
(f) Test the null hypothesis $\beta_{1}=1$ with $\alpha=0.05$.
(g) Give the $F$ statistic for testing the null hypothesis $\beta_{3}=0$.
(h) Give $S S R\left(x_{3} \mid x_{1}, x_{2}\right)$ and find $S S R\left(x_{3} \mid x_{1}, x_{2}, x_{4}\right)$.
(i) Test the model with only variables $x_{1}$ and $x_{2}$ against the model with all of variables $x_{1}, x_{2}, x_{3}$, and $x_{4}$.
(j) Test the model with only variables $x_{1}$ and $x_{2}$ against the model with variables $x_{1}, x_{2}$, and $x_{3}$.
(k) Should the test in part (g) be the same as the test in part (j)? Why or why not?
(l) For estimating the point on the regression surface at $\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=(100,25,50,4)$, the standard error of the estimate for the point on the surface is 2.62 . Give the estimated point on the surface, a $95 \%$ confidence interval for the point on the surface, and a $95 \%$ prediction interval for a new point with these $x$ values.
(m) Test the null hypothesis $\beta_{1}=\beta_{2}=\beta_{3}=\beta_{4}=0$ with $\alpha=0.05$.

