Math 1512 - Exam 1 Review - Fall 2025

Exam 1 covers §1.4-1.6, 1.8, 2.1-2.4 (unless told otherwise by your instructor)

Topics:

1. **Review Material**: Graph basic functions and their translations and transformations.
2. **Limits**: Evaluate $\lim_{x\to a} f(x)$, $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, given the graph of f or a formula for f. Understand infinite limits and be able to find vertical asymptotes. Be able to find limits using the Squeeze Theorem.
3. **Continuity**: Know the definition of continuity at a point. Use it to determine where a function is continuous.
4. **The Derivative**: Know the definition of the derivative as a limit. Use it to find derivatives. Recognize when a limit is a derivative. Know how graphs of functions and their derivatives are related. Use power, product, and quotient rules to find derivatives. Give examples of functions for which the derivative does not exist at some point $x = a$. Know how to differentiate all six trigonometric functions.
5. **Applications**: Find the equations of lines tangent or normal to a curve $y = f(x)$ at a point. Find points where the tangent line has zero slope.

Sample Problems:

1. Sketch the graphs of

   (a) $y = \sin(3x)$
   (b) $f(\theta) = 1-\cos(2\theta)$
   (c) $f(\theta) = 1 - 2\cos\theta$
   (d) $f(x) = \tan(2x)$
   (e) $f(x) = |x|$
   (f) $f(x) = |x - 1|$
   (g) $f(x) = 1/x$
   (h) $f(x) = 1/(x + 1)$
   (i) $f(x) = x + 1/x$
   (j) $f(x) = \sqrt{x + 1}$
   (k) $f(x) = x^2 - 2x$
   (l) $f(x) = x^2 - x^3$

2. Sketch the graphs of functions defined piecewise.
3. Evaluate the limits or explain why they do not exist.

   (a) $\lim_{x\to 0} \frac{x^2}{1 - x}$
   (b) $\lim_{x\to 1} \frac{x^2}{1 - x}$
   (c) $\lim_{x\to 1} \frac{1 - x^2}{1 - x}$
   (d) $\lim_{x\to 2} \frac{3x - 5\cos x}{x^2}$
   (e) $\lim_{x\to 0} \frac{3x - 5\cos x}{x^2}$
   (f) $\lim_{x\to -5} \frac{x^2 - 25}{x + 5}$

4. Evaluate the limits or explain why they do not exist, given $f(x) = \begin{cases} 1 + \sin x, & x < 0 \\ \cos x, & 0 \le x \le \pi \\ \sin x, & x > \pi \end{cases}$.

   (a) $\lim_{x\to 0^-} f(x)$
   (b) $\lim_{x\to 0^+} f(x)$
   (c) $\lim_{x\to 0} f(x)$
   (d) $\lim_{x\to \pi^-} f(x)$
   (e) $\lim_{x\to \pi^+} f(x)$
   (f) $\lim_{x\to \pi} f(x)$

5. Squeeze Theorem problems: §1.6: 38, 41
6. Continuity problems: §1.8: 23, 42, 44, 46, 47
7. State the definition of the derivative of a function $f(x)$ as a limit. Include a graph that illustrates the geometric interpretation of this limit.

8. For the following functions $f(x)$, find $f'(x)$ using the definition of the derivative as a limit. Confirm your result using the rules for differentiation.

   (a) $f(x) = \frac{x}{x+2}$ (b) $f(x) = \sqrt{2x}$ (c) $f(x) = x^4$

9. Find the derivatives of the following functions.

   (a) $f(x) = (3x+2)(x^2+3)$
   (b) $H(t) = \sqrt[3]{3t}(t+2) + \dfrac{1}{t^2\sqrt{t}}$
   (c) $f(x) = \dfrac{x^2+4x+3}{\sqrt{2x}}$
   (d) $h(t) = \dfrac{\sec t}{1-3t^2}$
   (e) $y = \dfrac{x}{x+1}$
   (f) $f(x) = \dfrac{1+\sin x}{1+\cos x}$
   (g) $f(x) = (2x-5)\tan x$
   (h) $g(t) = \sqrt{t}\cos t$
   (i) $f(u) = \dfrac{1-u^2}{1+u^2}$

10. The following limits all represent the slope of the line tangent to the graph of $y = f(x)$ at some point $x = a$, for some function f. In each case, state f and a. Evaluate the limits (*Hint: Just find* $f'(a)$).

    (a) $\lim\limits_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h}$ (b) $\lim\limits_{x\to 1} \dfrac{x^{17}-1}{x-1}$ (c) $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$ (d) $\lim\limits_{x\to 0} \dfrac{\cos(x)-1}{x}$

11. For each of the following functions: (i) sketch a graph of the function, (ii) use your sketch to sketch a graph of its derivative, (iii) find a formula for its derivative and check that the graph is consistent with your results in (ii)

    (a) $f(x) = x^{1/3}$ (b) $f(x) = \sec x$ (c) $f(x) = |x|x$

12. The gravitational force exerted by the earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \dfrac{GMr}{R^3} & \text{if } 0 \le r < R \\ \dfrac{GM}{r^2} & \text{if } r \ge R \end{cases}$$

    where M is the mass of the earth, R is its radius, and G is the gravitational constant (all three are positive constants).

    (a) Find $\lim\limits_{r\to R} F(r)$. Show you work clearly.
    (b) Determine where F is continuous.
    (c) Sketch a graph of $F(r)$ for $r > 0$.
    (d) Find a formula for the derivative $F'(r)$ and sketch a graph of F'.

13. (a) Find $p^{(5)}(x)$ if $p(x) = x^3 - 3x^2 + 2$.
    (b) Find $s^{(10)}(t)$ if $s(t) = \frac{1}{t}$.
    (c) How many nonzero derivatives can a polynomial of degree n have, at most?

14. §2.3: 84

15. Find equations of the lines tangent and normal to the curve $y = \sqrt{x} + x$ at the point (1,2).

16. Find the points on the curve $y = \dfrac{\cos x}{2+\sin x}$ in $[0, 2\pi]$ at which the tangent line is horizontal.