Math 1512 - Exam 2 Review - Fall 2025

Exam 2 covers §2.5-2.9, 3.1–3.5, 3.7; HW 4-8 (unless told otherwise by your instructor)

Topics and Sample Problems:

Chain Rule: Differentiate compositions of functions.

1. Differentiate the following functions:

   (a) $f(x) = \frac{x}{\sqrt{x^2+1}}$

   (b) $g(x) = x^2 \sin(x^3 - 3x)$

   (c) $h(t) = \cos(\sqrt{t})$

   (d) $q(t) = \sqrt{\cos(t)}$

2. §2.5: 78

Implicit Differentiation:

3. Find $\frac{dy}{dx}$ by implicit differentiation:

   (a) $x^3 - xy^2 + y^3 = 1$

   (b) $\sin(xy) = \cos(x + y)$

4. Find the equation of the tangent line to the following curve at the point $(\pi/2, \pi/4)$:

$$y \sin(2x) = x \cos(2y)$$

Rates of change: Interpret a derivative as an instantaneous rate of change, and find its units.

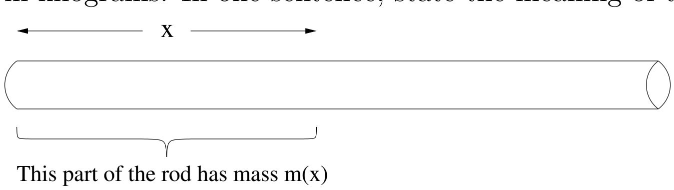
5. The total cost (in \$) of repaying a student loan at an interest rate of $r\%$ per year is $C = f(r)$.

   (a) What is the meaning of the derivative $f'(r)$? What are its units?

   (b) What does the statement $f'(10) = 1200$ mean?

   (c) Is $f'(r)$ always positive or does it change sign?

6. A rod of length L is shown in the figure below. Suppose the mass of the rod between its left endpoint and a point at distance x from it is $m(x), 0 \le x \le L$, where x is measured in cm and m is measured in kilograms. In one sentence, state the meaning of the derivative $m'(x)$. What are its units?



7. A particle moves along a straight line with position function $s(t) = t^3 - 9t^2 + 15t + 10, t \ge 0$, where t is measured in seconds and s is measured in feet.

   (a) Find the velocity at time t.

   (b) When is the particle at rest?

   (c) When is the particle moving in the negative direction?

   (d) When is the particle moving in the positive direction?

   (e) Find the total distance traveled during the first 8 s. (It may help to draw a diagram that illustrates the motion of the particle as in Figure 2, page 174.)

8. Suppose an object travels with constant velocity. What is its acceleration?

9. §2.7: 30 (violin string)

Related Rates: Given a relation between dependent variables, find a relation between their derivatives. Given a rate of change of one variable, find the rate of change of another (related) variable.

10. The pressure P, temperature T and volume V of a gas are related by the rule $T = cPV$ where c is a constant. At one instant the pressure of a gas is 30 KPa, the temperature is 18^oC, and the volume is 152 mL. If at that instant the temperature is increasing at 1^oC/hour and the volume is decreasing by 2mL/hour, at what rate is the pressure changing at that instant?

11. A rectangle is such that its length increases at the rate of 5 in/min and its width decreases at the rate of 7 in/min. At the moment when the length is 6 in and the width is 8 in, is the area increasing or decreasing?

12. §2.8: 29 (gravel pile)

13. §2.8: 39 (resistors)

Linearization: Approximate a function by linearization. Approximate a change of the function by a change of the linearization (or by using differentials).

14. §2.9: 5, 7 (just verify the linearization), 31, 35, 38, 40a, 41

Max/Min Values:

- Find absolute max/min of continuous functions on closed intervals: find critical numbers and compare function values at critical numbers and at endpoints.
- Find absolute max/min in general: find critical numbers and get enough information on increasing/decreasing or concavity to obtain a rough sketch of a graph that clearly illustrates absolute max/min.
- Find local max/min by finding critical numbers and using first or second derivative tests.

Graphing: Use intervals of increase/decrease, concave up/down, limiting behavior near vertical asymptotes, horizontal asymptotes, intercepts, domain, and symmetry to sketch the graph of a function. Be able to locate extreme values and inflection points.

15. Find domain, intervals where function is increasing/decreasing, intervals where function is concave up/down, and use behavior at infinity if applicable, to obtain a graph of the following functions. Find coordinates of all extrema and inflection points, and indicate them on the graph.

    (a) $f(x) = x^4 + 2x^3 + 2$

    (b) $f(x) = x - x^3$

    (c) $f(x) = x\sqrt{6-x}$

    (d) $f(x) = x + \dfrac{1}{x^2}$

    (e) $f(x) = x^2 + \dfrac{1}{x}$

    (f) $f(x) = x^{1/3}(x+4)$

16. §3.5: 47 (Hint: simplify the expression for the function by replacing $W/(24EI)$ by another letter, for example A.)

Optimization: These are word problems in which you need to identify the function that is to be maximized or minimized over a certain domain, and then use calculus to find the absolute max or min of that function on that particular domain.

17. §3.7: 9, 19, 20, 45, 51

18. Assume that if the price of a certain book is p dollars, then it will sell x copies where $x = 7000(1-p/35)$. Suppose the cost of producing those x copies is $15000+2.5x$ dollars. Finally, assume that the company will not sell this book for more than \$35. Determine the price for the book that will maximize profit.