Math 1512 - Exam 3 Review - Fall 2025

Exam 3 covers §3.9, 4.1-4.5, 5.1-5.2, 5.4-5.5; HW 9-12 (unless told otherwise by your instructor)

Antiderivatives/Indefinite Integrals, Differential Equations:

1. Find the most general antiderivative (i.e., indefinite integral) of a given function.
   Examples: §3.9: 5-23 odd

2. Find f given f' or f'' (with or without initial conditions). *Examples:* §3.9: 27-47 odd

3. Given the graph of a function, sketch the graph of the antiderivative (satisfying given conditions).
   Examples: §3.9: 53, 55

4. Given acceleration or velocity and initial conditions, find position. *Examples:* §3.9: 59-64

Definite Integrals:

5. Approximate integrals by Riemann sums, using either the right endpoint, left endpoint, or midpoint rule. *Example:* approximate $\int_0^\pi \sin\theta\, d\theta$ using four subintervals.

6. Rewrite integrals, using their definition, as limits of sums.
   Example: Write $\int_0^1 x\, dx$ as the limit of a sum (this is the definition of the integral).
   Answer: $\lim_{n\to\infty} \sum_{j=1}^{n} x_j \Delta x$ where $\{x_j\}, \Delta x$ is a uniform partition of $[0, 1]$.

7. Recognize when the limit of a sum is an integral and rewrite it as such. Then evaluate using the fundamental theorem. *Examples:*
   (a) Find $\lim_{n\to\infty} \sum_{j=1}^{n} \sin(x_j)\Delta x$ where $\{x_j\}, \Delta x$ is a uniform partition of $[0, \pi]$.
   (b) Write $\lim_{n\to\infty} \sum_{j=1}^{n} \sqrt{1 + f'(x_j)}\Delta x$, where $\{x_j\}$ is a uniform partition of $[a, b]$, as an integral.

8. Evaluate integrals by interpreting them as areas. *Examples:*
   (a) Find $\int_0^2 (2 - 3\sqrt{4 - x^2})\, dx$.
   (b) Find $\int_1^4 |x - 2|\, dx$.
   (c) Find $\int_0^2 H(x - 1)\, dx$ where H is the Heaviside function.

Fundamental Theorem of Calculus (FTC):

9. State the Fundamental Theorem of Calculus, part 1 and part 2.

10. Use the FTC part 1 (with chain rule if needed) to differentiate functions defined as integrals.
    Examples: §4.3: 9, 19, 61. Chapter 4 Review: 8bc, 59, 60.

11. Use the fundamental theorem of calculus (part 2) to evaluate integrals.
    Examples: Chapter 4 Review: 8a, 11-31 odd.

12. Understand the function $g(x) = \int_a^x f(t)\, dt$.
    Given the graph of f, can you sketch the graph of g? What is g'? Where is g increasing? Decreasing?
    Example: §4.3: 3

Rates of Change and Net Change Theorem:

13. If a quantity is given as an integral of a physical variable that has units, state the units of the integral. If a quantity is given as the derivative of a physical variable that has units, state the units of the derivative.

14. Given a rate of change of a quantity (e.g. velocity, flow rate, growth rate) find changes in the quantity.
    Examples: §4.4: 51-59, Chapter 4 Review, Concept Check: 5,6. Exercises: 53

15. **Substitution Rule**: Evaluate indefinite and definite integrals using substitution. (For definite integrals, don't forget to change the limits of integration.)

    Examples: §4.5: 1-51 odd

16. **Areas Between Curves**: Set up and evaluate a definite integral (with respect to x or y as appropriate) to compute the area between curves.

    Examples: §5.1: 1-33 odd

17. **Volumes**: Find volumes of solids of revolution, including problems where the function and region are not given (e.g. volume of a torus, spherical cap, sphere with a hole, etc.). Set up a definite integral (with respect to x or y as appropriate) using the method of slices (disks/washers), and evaluate the integral.

    Examples: §5.2: 1-27 odd, 61, 75, 84

18. **Work**: Set up and evaluate a definite integral to compute the work done by a force. Be able to use Hooke's law to compute work done when stretching or compressing a spring.

    Examples: §5.4: 1-12

19. **Average Value of a Function and MVT for Integrals**: Compute f_{avg}, the average value of a function $f(x)$ on some interval $[a, b]$. If possible, find c in $[a, b]$ such that $f(c) = f_{avg}$.

    Examples: §5.5: 1-10, 13-15, 17