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**HOMEWORK DAY 1** – *Tangent and velocity problems §1.4 (and §2.1 def 1-2)*

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1. You are given a function  $f$  and a point  $P$ . Express the slope of the tangent line to the graph of  $f$  at  $P$  as a limit of slopes of secant lines in two different ways: one as  $x \rightarrow a$ , and another as  $h \rightarrow 0$ . (see Definitions 1 and 2 in §2.1) Sketch a graph of the function showing the point  $P$  and a generic secant line through  $P$  and another point  $Q$  on the graph.

(a)  $f(x) = x^3$  at  $P(1, 1)$

(b)  $f(x) = \sqrt{x}$  at  $P(4, 2)$

2. The displacement (in meters) of a particle moving in a straight line is given by the equation of motion  $s(t) = 1/t^2$ , where  $t$  is measured in seconds.

(a) Express the velocity of the particle at  $t = 2$  as a limit.

(b) Sketch a graph of the function  $s(t)$ . What does the limit you found represent geometrically?

(c) What are the units of the velocity?

3. §1.4: 2

4. §1.4: 4

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**HOMEWORK DAY 2** – *The limit of a function. Infinite limits. §1.5*

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5. §1.5: 1

6. §1.5: 4. (a)

(b)

(c)

(d)

(e)

(f)

7. §1.5: 9. Add: (g)  $\lim_{x \rightarrow 6} f(x)$ .

(a)

(b)

(c)

(d)

(e)

(f)

(g)

8. Find the following limits. When a limit does not exist, briefly explain why.

(a) §1.5: 27    *Answer:*  $\lim_{x \rightarrow 5^+} \frac{x+1}{x-5} \left( \frac{6}{0^+} \right) + \infty$

(b) §1.5: 28

(c) §1.5: 29

(d) §1.5: 30

(e) §1.5: 31

(f) §1.5: 32

(g)  $\lim_{x \rightarrow 1^-} \frac{1}{x^3 - 1}$

$$(h) \lim_{x \rightarrow 1^+} \frac{1}{x^3 - 1}$$

$$(i) \lim_{x \rightarrow 1} \frac{1}{x^3 - 1}$$

9. §1.5: 11

10. §1.5: 16

11. Let  $f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2-x & \text{if } 1 < x \leq 3 \\ -1 & \text{if } x > 3 \end{cases}$

(a) Sketch a graph of the function.

(b) Find the following limits or determine they do not exist (if so, explain why not).

$$\lim_{x \rightarrow -2} f(x)$$

$$\lim_{x \rightarrow -1} f(x)$$

$$\lim_{x \rightarrow 1} f(x)$$

$$\lim_{x \rightarrow 3} f(x)$$

(c) State the values of all  $a$  for which  $\lim_{x \rightarrow a} f(x)$  exists.

12. Consider the two functions  $f(x) = \frac{x^2-x}{x-1}$  and  $g(x) = x$ .

(a) Are the two functions equal? Explain.

(b) Sketch a graph of both functions.

(c) Use your sketch to find  $\lim_{x \rightarrow 1} f(x)$  and  $\lim_{x \rightarrow 1} g(x)$ . Illustrate the limit in your sketch.

13. Consider the function  $g_1(x) = \frac{x^2 - 4}{x - 2}$ .

(a) Sketch the graph of  $g_1(x)$  and of  $g_2(x) = x + 2$ .

(b) Find  $\lim_{x \rightarrow 2} g_1(x)$  and  $\lim_{x \rightarrow 2} g_2(x)$

(c) Explain why the limit in (b) is the slope of tangent line of  $f(x) = x^2$  at  $x = 2$ . Illustrate with a figure.

14. Let  $f(x) = \sin(x)$ .

(a) Sketch a clearly labeled graph of  $f(x)$ , using a 1-1 scale.

(b) Explain why the slope of the tangent line to the graph of  $f$  at the origin ( $x = 0$ ) is given by the limit

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} .$$

(c) Approximate the limit using a table of values.

(d) Use your result in (c) to find an equation for the tangent line to  $f$  at the origin, and add a graph of it to your sketch in (a).

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**HOMEWORK DAY 3** – *Finding limits using limit laws. §1.6*

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15. Find the following finite or infinite limits. If the limit does not exist, explain why not. Follow the worked out example.

(a)  $\lim_{x \rightarrow -2} (2x + x^2)$

(b)  $\lim_{h \rightarrow 1} (2 - h/2)$

(c)  $\lim_{x \rightarrow 1^+} \frac{1}{x-1}$     *Answer:*  $\lim_{x \rightarrow 1^+} \frac{1}{x-1} \left( \frac{1}{0^+} \right) + \infty$

(d)  $\lim_{x \rightarrow -3^-} \frac{x+2}{x+3}$

(e)  $\lim_{x \rightarrow 1} \frac{1}{x-1}$

(f)  $\lim_{x \rightarrow 1} \frac{1-2x}{(x-1)^2}$

$$(g) \lim_{x \rightarrow \pi} \frac{1 + \cos x}{1 - x}$$

$$(h) \lim_{x \rightarrow \pi} \frac{1 - x}{1 + \cos x}$$

$$(i) \lim_{t \rightarrow 1} \frac{1 - t^2}{1 - t}$$

$$(j) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

$$(k) \lim_{x \rightarrow 1} \frac{(1+x)^3 - 1}{x}$$

(l)  $\lim_{x \rightarrow 0} \frac{(1+x)^3 - 1}{x}$

(m) §1.6: 13

(n) §1.6: 17

(o) §1.6: 19

(p) §1.6: 21

(q) §1.6: 23

(r) §1.6: 25

(s) §1.6: 27

16. §1.6: 38 (Squeeze Theorem)

17. §1.6: 41 (Squeeze Theorem)