HOMEWORK DAY 18 – *Derivatives and the shape of the graph §3.3*

1. §3.3: 1

2. §3.3: 8

3. §3.3: 10

4. For the function $f(x) = 3x^4 - 4x^3 + 2$

   (a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

   (b) Find the intervals where the function is concave up/down and the coordinates at all inflection points.

(c) Use the above information to sketch a rough graph of the function.

5. For the function $f(x) = \cos^2 x - 2\sin x,\ x \in [0, 2\pi]$

   (a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

(b) Find the intervals where the function is concave up/down and the coordinates at all inflection points.

(c) Use the above information to sketch a rough graph of the function.

6. §3.4: 4

7. Find the following limits. Carefully show your work.

   (a) §3.4: 9

   (b) §3.4: 11

   (c) §3.4: 12

8. For the function $f(x) = \dfrac{2}{x^2 + 4}$

(a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

(b) Find the intervals where the function is concave up/down and the coordinates of all inflection points.

(c) Find the limiting behaviour as x approaches $\pm\infty$.

(d) Use the above information to sketch a rough graph of the function. Include all intercepts on graph.

9. Let $f(x) = \dfrac{4 - x}{3 + x}$.

(a) Find the intercepts of f and all its asymptotes.
Find the limiting behaviour near each vertical asymptotes ($\lim_{x \to a^{\pm}} f(x)$).
Use this information to sketch a guess for the graph of f.

(b) Confirm details of your guess by finding the intervals where the function is increasing/decreasing and concave up/down. Clearly mark any local extrema and inflection points in your plot.

(c) Use algebra (long division in this case) to rewrite the function as a polynomial plus a proper quotient. Could you have obtained the graph of f from this result?

10. The reaction rate V of a common enzyme reaction is given in terms of substrate level S by

$$V = \frac{V_o S}{K + S}, \quad S \geq 0$$

where V_o and K are positive constants.

(a) Show that V is an increasing function of S. Explain why it follows that V has no absolute maximum value.

(b) What is $\lim_{S \to \infty} V$?

(c) Determine the concavity of the graph of V (where is it concave up? where concave down?).

(d) Sketch a graph of V as a function of S that reflects all the information you found above.