**HOMEWORK DAY 26** – *Antiderivatives §3.9*

1. Find the most general antiderivative for the following functions.

   (a) $f(x) = 6x^5 - 8x^4 - 9x^2 - 3$

   (b) $f(x) = 3x^4 - \frac{2}{3}x^3 + \pi x + \sqrt{2}$

   (c) $f(x) = 4x^{-2/3} - 2x^{5/3}, \quad x > 0$

   (d) $f(x) = (x-5)^2$

   (e) $f(x) = \sin(4x)$

(f) $f(t) = \sqrt[4]{t} + \sqrt[4]{x}, \quad t, x > 0$

(g) $f(s) = \dfrac{4 - 2s + \sqrt{s}}{s^{1/2}}, \quad s > 0$

2. Find $f$.

   (a) $f'(x) = \sqrt{x} - 2$ , $\quad f(9) = 4$

   (b) $f'(x) = 5x^{2/3}$ , $\quad f(8) = 21$

(c) $f'(t) = t + \dfrac{1}{t^3}$ , $\quad t > 0$ , $\quad f(1) = 6$

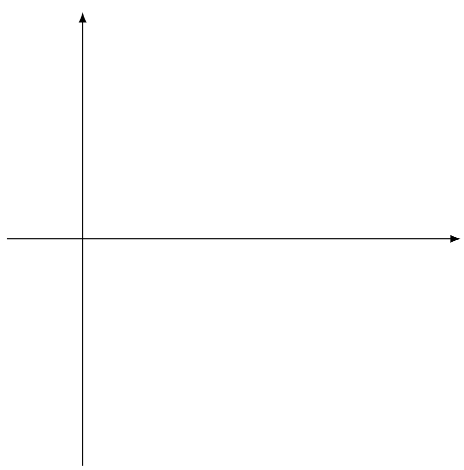
(d) $f'(\theta) = \sec\theta(\sec\theta + \tan\theta)$ , $\quad f(\pi/4) = 1, \quad \theta \in (-\pi/2, \pi/2)$

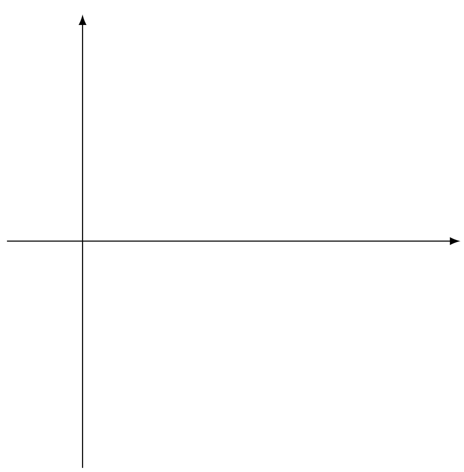
(e) $f''(x) = 8x^3 + 5$ , $\quad f(1) = 0$ , $\quad f'(1) = 8$

(f) $f''(t) = \cos t + \sin t$ ,  $f(0) = 3$ ,  $f'(0) = 4$

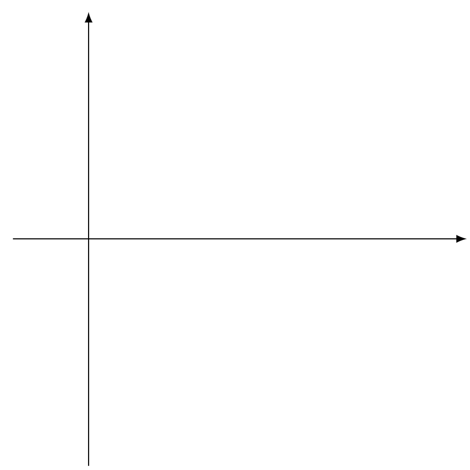
3. Draw the plots of the two functions one below the other. (Label all axes.)

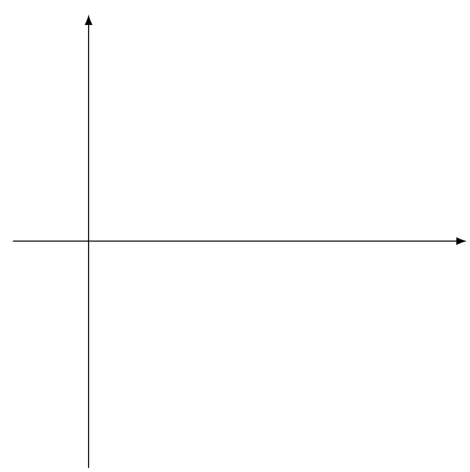
(a) §3.9: 53





(b) 3.9: 55





4. §4.1: 2

5. Approximate the area under the graph of $f(x) = \sin x$ between $x = 0$ and $x = \pi$ using six rectangles and left endpoints to determine the height of each rectangle. Simplify your answer as much as possible without using a calculator. Sketch a graph of $f(x)$ and the six rectangles.

6. §4.1: 13

7. Evaluate the following sums.

(a) $\sum_{k=2}^{5} \frac{2k}{k-1}$

(b) $\sum_{j=0}^{5} j^2 \sin(j\pi/6)$

(c) $\sum_{k=3}^{100} 2$

(d) $\sum_{j=1}^{1000} j$

(Hint: see Formula 6 in §4.2)

8. Use summation notation to express the sums

   (a) $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

   (b) $3 + 6 + 9 + 12 + 15$

   (c) $2/5 + 3/7 + 4/9 + 5/11$

9. Which is larger, $\sum_{j=1}^{N} j^2$ or $\sum_{j=1}^{N^2} j$ ? Explain why.

10. *The definite integral is the limit of a Riemann sum.*

    (a) §4.2: 20 (express limit as a definite integral)

    (b) §4.2: 26 (express definite integral as a limit of a Riemann sum)

    (c) §4.2: 84 (express limit as a definite integral)

11. *Approximating and evaluating the limit.* Consider the integral $\int_0^3 x^2\,dx$.

    (a) Use a Riemann sum with $n = 6$ subintervals and right endpoints to approximate the integral. Round your answer to four decimal places. Include a graph

(b) Express the integral as the limit of a Riemann sum.

(c) Evaluate the limit in (b). *Hint:* $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

12. *Interpreting the definite integral in terms of areas.*

(a) Consider the function $g(x)$ given in §4.2: 36. Evaluate

i. $\int_0^2 g(x)\,dx =$

ii. $\int_2^6 g(x)\,dx =$

iii. $\int_0^7 g(x)\,dx =$

iv. $\int_0^7 |g(x)|\,dx =$

(b) §4.2: 42

(c) §4.2: 44

(d) §4.2: 46

(e) §4.2: 63

13. Using the properties of the definite integral.

    (a) §4.2: 51

    (b) §4.2: 52

    (c) §4.2: 57

    (d) §4.2: 58

The **Heaviside function** H is defined by $H(t) = \begin{cases} 0\ , & t < 0 \\ 1\ , & t \geq 0 \end{cases}$

(a) Sketch $H(t)$. Find $\int_{-2}^{4} H(t)\, dt$.

(b) Sketch $H(t-2)$. Find $\int_{-2}^{4} H(t-2)\, dt$.

(c) Sketch $H(t-2) + H(t)$ (use superposition of graphs).

Use the above to find $\int_{-2}^{4} [H(t-2) + H(t)]\, dt$