

Diagnostic Tests

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A: Diagnostic Test: Algebra

1. Evaluate each expression without using a calculator.

a. $(-3)^4$

b. -3^4

c. 3^{-4}

d. $\frac{5^{23}}{5^{21}}$

e. $\left(\frac{2}{3}\right)^{-2}$

f. $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

a. $\sqrt{200} - \sqrt{32}$

b. $(3a^3b^3)(4ab^2)^2$

c. $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

a. $3(x + 6) + 4(2x - 5)$

b. $(x + 3)(4x - 5)$

c. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

d. $(2x + 3)^2$

e. $(x + 2)^3$

4. Factor each expression.

a. $4x^2 - 25$

b. $2x^2 + 5x - 12$

c. $x^3 - 3x^2 - 4x + 12$

d. $x^4 + 27x$

e. $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

f. $x^3y - 4xy$

5. Simplify the rational expression.

a. $\frac{x^2 + 3x + 2}{x^2 - x - 2}$

b. $\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x + 3}{2x + 1}$

c. $\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2}$

d. $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}$

6. Rationalize the expression and simplify.

a. $\frac{\sqrt{10}}{\sqrt{5} - 2}$

b. $\frac{\sqrt{4+h} - 2}{h}$

7. Rewrite by completing the square.

a. $x^2 + x + 1$

b. $2x^2 - 12x + 11$

8. Solve the equation. (Find only the real solutions.)

a. $x + 5 = 14 - \frac{1}{2}x$

b. $\frac{2x}{x+1} = \frac{2x-1}{x}$

c. $x^2 - x - 12 = 0$

d. $2x^2 + 4x + 1 = 0$

e. $x^4 - 3x^2 + 2 = 0$

f. $3|x - 4| = 10$

g. $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$

9. Solve each inequality. Write your answer using interval notation.

a. $-4 < 5 - 3x \leq 17$

b. $x^2 < 2x + 8$

c. $x(x-1)(x+2) > 0$

d. $|x - 4| < 3$

e. $\frac{2x-3}{x+1} \leq 1$

10. State whether each equation is true or false.

a. $(p+q)^2 = p^2 + q^2$

b. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

c. $\sqrt{a^2 + b^2} = a + b$

d. $\frac{1+TC}{C} = 1+T$

e. $\frac{1}{x-y} = \frac{1}{x} - \frac{1}{y}$

f. $\frac{1/x}{a/x - b/x} = \frac{1}{a-b}$

If you had difficulty with these problems, you may wish to consult the Review of Algebra on the website StewartCalculus.com.

B: Diagnostic Test: Analytic Geometry

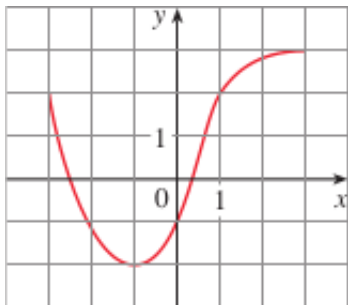
- Find an equation for the line that passes through the point $(2, -5)$ and
 - has slope -3
 - is parallel to the x -axis
 - is parallel to the y -axis
 - is parallel to the line $2x - 4y = 3$
- Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.
- Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.
- Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
 - Find the slope of the line that contains A and B .
 - Find an equation of the line that passes through A and B . What are the intercepts?
 - Find the midpoint of the segment AB .
 - Find the length of the segment AB .
 - Find an equation of the perpendicular bisector of AB .

- f. Find an equation of the circle for which AB is a diameter.
5. Sketch the region in the xy -plane defined by the equation or inequalities.
- $-1 \leq y \leq 3$
 - $|x| < 4$ and $|y| < 2$
 - $y < 1 - \frac{1}{2}x$
 - $y \geq x^2 - 1$
 - $x^2 + y^2 < 4$
 - $9x^2 + 16y^2 = 144$

If you had difficulty with these problems, you may wish to consult the review of analytic geometry in [Appendixes B and C](#).

C: Diagnostic Test: Functions

1. The [graph](#) of a function f is given below.



- State the value of $f(-1)$.
- Estimate the value of $f(2)$.
- For what values of x is $f(x) = 2$?
- Estimate the values of x such that $f(x) = 0$.
- State the domain and range of f .

2. If $f(x) = x^3$, evaluate the difference quotient $\frac{f(2+h) - f(2)}{h}$ and simplify your answer.
3. Find the domain of the function.
- $f(x) = \frac{2x + 1}{x^2 + x - 2}$
 - $g(x) = \frac{\sqrt[3]{x}}{x^2 + 1}$
 - $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$
4. How are graphs of the functions obtained from the graph of f ?
- $y = -f(x)$
 - $y = 2f(x) - 1$
 - $y = f(x - 3) + 2$
5. Without using a calculator, make a rough sketch of the graph.
- $y = x^3$
 - $y = (x + 1)^3$
 - $y = (x - 2)^3 + 3$
 - $y = 4 - x^2$
 - $y = \sqrt{x}$
 - $y = 2\sqrt{x}$
 - $y = -2^x$
 - $y = 1 + x^{-1}$
6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$
- Evaluate $f(-2)$ and $f(1)$.
 - Sketch the graph of f .

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

a. $f \circ g$

b. $g \circ f$

c. $g \circ g \circ g$

If you had difficulty with these problems, you should look at [sections 1.1](#), [1.2](#), and [1.3](#) of this book.

D: Diagnostic Test: Trigonometry

1. Convert from degrees to radians.

a. 300°

b. -18°

2. Convert from radians to degrees.

a. $5\pi/6$

b. 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of 30° .

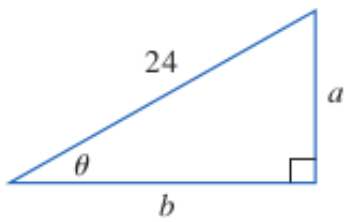
4. Find the exact values.

a. $\tan(\pi/3)$

b. $\sin(7\pi/6)$

c. $\sec(5\pi/3)$

5. Express the lengths a and b in the [figure](#) in terms of θ .



6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.
7. Prove the identities.
- a. $\tan \theta \sin \theta + \cos \theta = \sec \theta$
- b. $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$
8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$.
9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

If you had difficulty with these problems, you should look at [Appendix D](#) of this book.

□ DIAGNOSTIC TESTS SOLUTIONS

Test A Algebra

1. (a) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

(b) $-3^4 = -(3)(3)(3)(3) = -81$

(c) $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

(d) $\frac{5^{23}}{5^{21}} = 5^{23-21} = 5^2 = 25$

(e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(f) $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$

2. (a) Note that $\sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$ and $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$. Thus $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$.

(b) $(3a^3b^3)(4ab^2)^2 = 3a^3b^3 \cdot 16a^2b^4 = 48a^5b^7$

(c) $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(\frac{x^2y^{-1/2}}{3x^{3/2}y^3}\right)^2 = \frac{(x^2y^{-1/2})^2}{(3x^{3/2}y^3)^2} = \frac{x^4y^{-1}}{9x^3y^6} = \frac{x^4}{9x^3y^6y} = \frac{x}{9y^7}$

3. (a) $3(x+6) + 4(2x-5) = 3x + 18 + 8x - 20 = 11x - 2$

(b) $(x+3)(4x-5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} - (\sqrt{b})^2 = a - b$

Or: Use the formula for the difference of two squares to see that $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$.

(d) $(2x+3)^2 = (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$.

Note: A quicker way to expand this binomial is to use the formula $(a+b)^2 = a^2 + 2ab + b^2$ with $a = 2x$ and $b = 3$:

$$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$$

(e) See Reference Page 1 for the binomial formula $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Using it, we get

$$(x+2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3 = x^3 + 6x^2 + 12x + 8.$$

4. (a) Using the difference of two squares formula, $a^2 - b^2 = (a+b)(a-b)$, we have

$$4x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5).$$

(b) Factoring by trial and error, we get $2x^2 + 5x - 12 = (2x-3)(x+4)$.

(c) Using factoring by grouping and the difference of two squares formula, we have

$$x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x^2-4)(x-3) = (x-2)(x+2)(x-3).$$

(d) $x^4 + 27x = x(x^3 + 27) = x(x+3)(x^2 - 3x + 9)$

This last expression was obtained using the sum of two cubes formula, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ with $a = x$ and $b = 3$. [See Reference Page 1 in the textbook.]

(e) The smallest exponent on x is $-\frac{1}{2}$, so we will factor out $x^{-1/2}$.

$$3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x^2 - 3x + 2) = 3x^{-1/2}(x-1)(x-2)$$

(f) $x^3y - 4xy = xy(x^2 - 4) = xy(x-2)(x+2)$

2 □ DIAGNOSTIC TESTS

5. (a) $\frac{x^2 + 3x + 2}{x^2 - x - 2} = \frac{(x+1)(x+2)}{(x+1)(x-2)} = \frac{x+2}{x-2}$
- (b) $\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x+3}{2x+1} = \frac{(2x+1)(x-1)}{(x-3)(x+3)} \cdot \frac{x+3}{2x+1} = \frac{x-1}{x-3}$
- (c) $\frac{x^2}{x^2 - 4} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} \cdot \frac{x-2}{x-2} = \frac{x^2 - (x+1)(x-2)}{(x-2)(x+2)}$
 $= \frac{x^2 - (x^2 - x - 2)}{(x+2)(x-2)} = \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$
- (d) $\frac{\frac{y}{1} - \frac{x}{1}}{\frac{y}{1} - \frac{x}{1}} = \frac{\frac{y}{1} - \frac{x}{1}}{\frac{y}{1} - \frac{x}{1}} \cdot \frac{xy}{xy} = \frac{y^2 - x^2}{x - y} = \frac{(y-x)(y+x)}{-(y-x)} = \frac{y+x}{-1} = -(x+y)$
6. (a) $\frac{\sqrt{10}}{\sqrt{5}-2} = \frac{\sqrt{10}}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{50} + 2\sqrt{10}}{(\sqrt{5})^2 - 2^2} = \frac{5\sqrt{2} + 2\sqrt{10}}{5-4} = 5\sqrt{2} + 2\sqrt{10}$
- (b) $\frac{\sqrt{4+h}-2}{h} = \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \frac{4+h-4}{h(\sqrt{4+h}+2)} = \frac{h}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$
7. (a) $x^2 + x + 1 = (x^2 + x + \frac{1}{4}) + 1 - \frac{1}{4} = (x + \frac{1}{2})^2 + \frac{3}{4}$
- (b) $2x^2 - 12x + 11 = 2(x^2 - 6x) + 11 = 2(x^2 - 6x + 9 - 9) + 11 = 2(x^2 - 6x + 9) - 18 + 11 = 2(x-3)^2 - 7$
8. (a) $x + 5 = 14 - \frac{1}{2}x \Leftrightarrow x + \frac{1}{2}x = 14 - 5 \Leftrightarrow \frac{3}{2}x = 9 \Leftrightarrow x = \frac{2}{3} \cdot 9 \Leftrightarrow x = 6$
- (b) $\frac{2x}{x+1} = \frac{2x-1}{x} \Rightarrow 2x^2 = (2x-1)(x+1) \Leftrightarrow 2x^2 = 2x^2 + x - 1 \Leftrightarrow x = 1$
- (c) $x^2 - x - 12 = 0 \Leftrightarrow (x+3)(x-4) = 0 \Leftrightarrow x+3 = 0 \text{ or } x-4 = 0 \Leftrightarrow x = -3 \text{ or } x = 4$
- (d) By the quadratic formula, $2x^2 + 4x + 1 = 0 \Leftrightarrow$
 $x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{2(-2 \pm \sqrt{2})}{4} = \frac{-2 \pm \sqrt{2}}{2} = -1 \pm \frac{1}{2}\sqrt{2}$.
- (e) $x^4 - 3x^2 + 2 = 0 \Leftrightarrow (x^2 - 1)(x^2 - 2) = 0 \Leftrightarrow x^2 - 1 = 0 \text{ or } x^2 - 2 = 0 \Leftrightarrow x^2 = 1 \text{ or } x^2 = 2 \Leftrightarrow$
 $x = \pm 1 \text{ or } x = \pm\sqrt{2}$
- (f) $3|x-4| = 10 \Leftrightarrow |x-4| = \frac{10}{3} \Leftrightarrow x-4 = -\frac{10}{3} \text{ or } x-4 = \frac{10}{3} \Leftrightarrow x = \frac{2}{3} \text{ or } x = \frac{22}{3}$
- (g) Multiplying through $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$ by $(4-x)^{1/2}$ gives $2x - 3(4-x) = 0 \Leftrightarrow$
 $2x - 12 + 3x = 0 \Leftrightarrow 5x - 12 = 0 \Leftrightarrow 5x = 12 \Leftrightarrow x = \frac{12}{5}$.
9. (a) $-4 < 5 - 3x \leq 17 \Leftrightarrow -9 < -3x \leq 12 \Leftrightarrow 3 > x \geq -4 \text{ or } -4 \leq x < 3$.
 In interval notation, the answer is $[-4, 3)$.
- (b) $x^2 < 2x + 8 \Leftrightarrow x^2 - 2x - 8 < 0 \Leftrightarrow (x+2)(x-4) < 0$. Now, $(x+2)(x-4)$ will change sign at the critical values $x = -2$ and $x = 4$. Thus the possible intervals of solution are $(-\infty, -2)$, $(-2, 4)$, and $(4, \infty)$. By choosing a single test value from each interval, we see that $(-2, 4)$ is the only interval that satisfies the inequality.

(c) The inequality $x(x-1)(x+2) > 0$ has critical values of $-2, 0,$ and 1 . The corresponding possible intervals of solution are $(-\infty, -2), (-2, 0), (0, 1)$ and $(1, \infty)$. By choosing a single test value from each interval, we see that both intervals $(-2, 0)$ and $(1, \infty)$ satisfy the inequality. Thus, the solution is the union of these two intervals: $(-2, 0) \cup (1, \infty)$.

(d) $|x-4| < 3 \Leftrightarrow -3 < x-4 < 3 \Leftrightarrow 1 < x < 7$. In interval notation, the answer is $(1, 7)$.

(e) $\frac{2x-3}{x+1} \leq 1 \Leftrightarrow \frac{2x-3}{x+1} - 1 \leq 0 \Leftrightarrow \frac{2x-3}{x+1} - \frac{x+1}{x+1} \leq 0 \Leftrightarrow \frac{2x-3-x-1}{x+1} \leq 0 \Leftrightarrow \frac{x-4}{x+1} \leq 0$.

Now, the expression $\frac{x-4}{x+1}$ may change signs at the critical values $x = -1$ and $x = 4$, so the possible intervals of solution are $(-\infty, -1), (-1, 4],$ and $[4, \infty)$. By choosing a single test value from each interval, we see that $(-1, 4]$ is the only interval that satisfies the inequality.

10. (a) False. In order for the statement to be true, it must hold for all real numbers, so, to show that the statement is false, pick $p = 1$ and $q = 2$ and observe that $(1+2)^2 \neq 1^2 + 2^2$. In general, $(p+q)^2 = p^2 + 2pq + q^2$.

(b) True as long as a and b are nonnegative real numbers. To see this, think in terms of the laws of exponents:

$$\sqrt{ab} = (ab)^{1/2} = a^{1/2}b^{1/2} = \sqrt{a}\sqrt{b}.$$

(c) False. To see this, let $p = 1$ and $q = 2$, then $\sqrt{1^2+2^2} \neq 1+2$.

(d) False. To see this, let $T = 1$ and $C = 2$, then $\frac{1+1(2)}{2} \neq 1+1$.

(e) False. To see this, let $x = 2$ and $y = 3$, then $\frac{1}{2-3} \neq \frac{1}{2} - \frac{1}{3}$.

(f) True since $\frac{1/x}{a/x - b/x} \cdot \frac{x}{x} = \frac{1}{a-b}$, as long as $x \neq 0$ and $a-b \neq 0$.

Test B Analytic Geometry

1. (a) Using the point $(2, -5)$ and $m = -3$ in the point-slope equation of a line, $y - y_1 = m(x - x_1)$, we get

$$y - (-5) = -3(x - 2) \Rightarrow y + 5 = -3x + 6 \Rightarrow y = -3x + 1.$$

(b) A line parallel to the x -axis must be horizontal and thus have a slope of 0. Since the line passes through the point $(2, -5)$, the y -coordinate of every point on the line is -5 , so the equation is $y = -5$.

(c) A line parallel to the y -axis is vertical with undefined slope. So the x -coordinate of every point on the line is 2 and so the equation is $x = 2$.

(d) Note that $2x - 4y = 3 \Rightarrow -4y = -2x + 3 \Rightarrow y = \frac{1}{2}x - \frac{3}{4}$. Thus the slope of the given line is $m = \frac{1}{2}$. Hence, the slope of the line we're looking for is also $\frac{1}{2}$ (since the line we're looking for is required to be parallel to the given line).

$$\text{So the equation of the line is } y - (-5) = \frac{1}{2}(x - 2) \Rightarrow y + 5 = \frac{1}{2}x - 1 \Rightarrow y = \frac{1}{2}x - 6.$$

2. First we'll find the distance between the two given points in order to obtain the radius, r , of the circle:

$$r = \sqrt{[3 - (-1)]^2 + (-2 - 4)^2} = \sqrt{4^2 + (-6)^2} = \sqrt{52}.$$

Next use the standard equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center, to get $(x + 1)^2 + (y - 4)^2 = 52$.

4 □ DIAGNOSTIC TESTS

3. We must rewrite the equation in standard form in order to identify the center and radius. Note that

$x^2 + y^2 - 6x + 10y + 9 = 0 \Rightarrow x^2 - 6x + 9 + y^2 + 10y = 0$. For the left-hand side of the latter equation, we factor the first three terms and complete the square on the last two terms as follows: $x^2 - 6x + 9 + y^2 + 10y = 0 \Rightarrow (x - 3)^2 + y^2 + 10y + 25 = 25 \Rightarrow (x - 3)^2 + (y + 5)^2 = 25$. Thus, the center of the circle is $(3, -5)$ and the radius is 5.

4. (a) $A(-7, 4)$ and $B(5, -12) \Rightarrow m_{AB} = \frac{-12 - 4}{5 - (-7)} = \frac{-16}{12} = -\frac{4}{3}$

(b) $y - 4 = -\frac{4}{3}[x - (-7)] \Rightarrow y - 4 = -\frac{4}{3}x - \frac{28}{3} \Rightarrow 3y - 12 = -4x - 28 \Rightarrow 4x + 3y + 16 = 0$. Putting $y = 0$, we get $4x + 16 = 0$, so the x -intercept is -4 , and substituting 0 for x results in a y -intercept of $-\frac{16}{3}$.

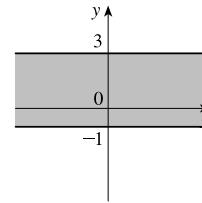
(c) The midpoint is obtained by averaging the corresponding coordinates of both points: $(\frac{-7+5}{2}, \frac{4+(-12)}{2}) = (-1, -4)$.

(d) $d = \sqrt{[5 - (-7)]^2 + (-12 - 4)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

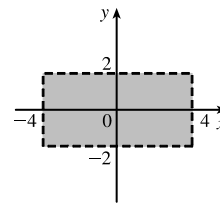
(e) The perpendicular bisector is the line that intersects the line segment \overline{AB} at a right angle through its midpoint. Thus the perpendicular bisector passes through $(-1, -4)$ and has slope $\frac{3}{4}$ [the slope is obtained by taking the negative reciprocal of the answer from part (a)]. So the perpendicular bisector is given by $y + 4 = \frac{3}{4}[x - (-1)]$ or $3x - 4y = 13$.

(f) The center of the required circle is the midpoint of \overline{AB} , and the radius is half the length of \overline{AB} , which is 10. Thus, the equation is $(x + 1)^2 + (y + 4)^2 = 100$.

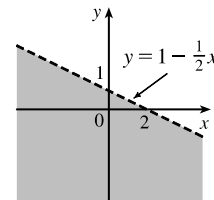
5. (a) Graph the corresponding horizontal lines (given by the equations $y = -1$ and $y = 3$) as solid lines. The inequality $y \geq -1$ describes the points (x, y) that lie on or *above* the line $y = -1$. The inequality $y \leq 3$ describes the points (x, y) that lie on or *below* the line $y = 3$. So the pair of inequalities $-1 \leq y \leq 3$ describes the points that lie on or *between* the lines $y = -1$ and $y = 3$.



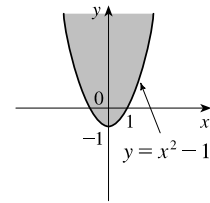
(b) Note that the given inequalities can be written as $-4 < x < 4$ and $-2 < y < 2$, respectively. So the region lies between the vertical lines $x = -4$ and $x = 4$ and between the horizontal lines $y = -2$ and $y = 2$. As shown in the graph, the region common to both graphs is a rectangle (minus its edges) centered at the origin.



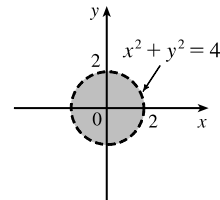
(c) We first graph $y = 1 - \frac{1}{2}x$ as a dotted line. Since $y < 1 - \frac{1}{2}x$, the points in the region lie *below* this line.



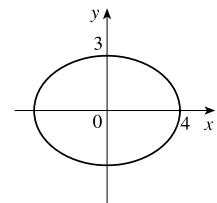
- (d) We first graph the parabola $y = x^2 - 1$ using a solid curve. Since $y \geq x^2 - 1$, the points in the region lie on or *above* the parabola.



- (e) We graph the circle $x^2 + y^2 = 4$ using a dotted curve. Since $\sqrt{x^2 + y^2} < 2$, the region consists of points whose distance from the origin is less than 2, that is, the points that lie *inside* the circle.



- (f) The equation $9x^2 + 16y^2 = 144$ is an ellipse centered at $(0, 0)$. We put it in standard form by dividing by 144 and get $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The x -intercepts are located at a distance of $\sqrt{16} = 4$ from the center while the y -intercepts are a distance of $\sqrt{9} = 3$ from the center (see the graph).



Test C Functions

- Locate -1 on the x -axis and then go down to the point on the graph with an x -coordinate of -1 . The corresponding y -coordinate is the value of the function at $x = -1$, which is -2 . So, $f(-1) = -2$.
 - Using the same technique as in part (a), we get $f(2) \approx 2.8$.
 - Locate 2 on the y -axis and then go left and right to find all points on the graph with a y -coordinate of 2 . The corresponding x -coordinates are the x -values we are searching for. So $x = -3$ and $x = 1$.
 - Using the same technique as in part (c), we get $x \approx -2.5$ and $x \approx 0.3$.
 - The domain is all the x -values for which the graph exists, and the range is all the y -values for which the graph exists. Thus, the domain is $[-3, 3]$, and the range is $[-2, 3]$.
- Note that $f(2 + h) = (2 + h)^3$ and $f(2) = 2^3 = 8$. So the difference quotient becomes

$$\frac{f(2 + h) - f(2)}{h} = \frac{(2 + h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h} = \frac{h(12 + 6h + h^2)}{h} = 12 + 6h + h^2.$$
- Set the denominator equal to 0 and solve to find restrictions on the domain: $x^2 + x - 2 = 0 \Rightarrow (x - 1)(x + 2) = 0 \Rightarrow x = 1$ or $x = -2$. Thus, the domain is all real numbers except 1 or -2 or, in interval notation, $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.
 - Note that the denominator is always greater than or equal to 1 , and the numerator is defined for all real numbers. Thus, the domain is $(-\infty, \infty)$.
 - Note that the function h is the sum of two root functions. So h is defined on the intersection of the domains of these two root functions. The domain of a square root function is found by setting its radicand greater than or equal to 0 . Now,

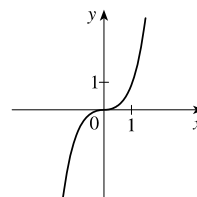
6 □ DIAGNOSTIC TESTS

$4 - x \geq 0 \Rightarrow x \leq 4$ and $x^2 - 1 \geq 0 \Rightarrow (x - 1)(x + 1) \geq 0 \Rightarrow x \leq -1$ or $x \geq 1$. Thus, the domain of h is $(-\infty, -1] \cup [1, 4]$.

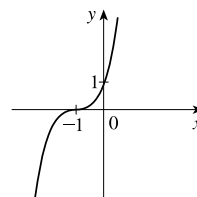
4. (a) Reflect the graph of f about the x -axis.
 (b) Stretch the graph of f vertically by a factor of 2, then shift 1 unit downward.
 (c) Shift the graph of f right 3 units, then up 2 units.

5. (a) Make a table and then connect the points with a smooth curve:

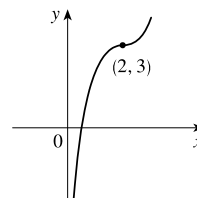
x	-2	-1	0	1	2
y	-8	-1	0	1	8



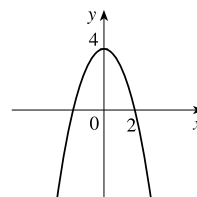
- (b) Shift the graph from part (a) left 1 unit.



- (c) Shift the graph from part (a) right 2 units and up 3 units.

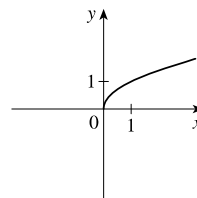


- (d) First plot $y = x^2$. Next, to get the graph of $f(x) = 4 - x^2$, reflect f about the x -axis and then shift it upward 4 units.

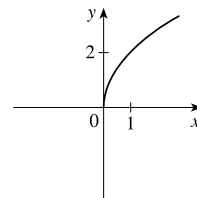


- (e) Make a table and then connect the points with a smooth curve:

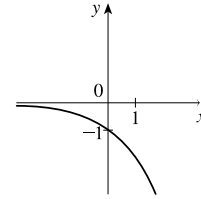
x	0	1	4	9
y	0	1	2	3



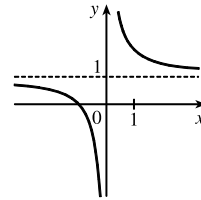
- (f) Stretch the graph from part (e) vertically by a factor of two.



- (g) First plot $y = 2^x$. Next, get the graph of $y = -2^x$ by reflecting the graph of $y = 2^x$ about the x -axis.

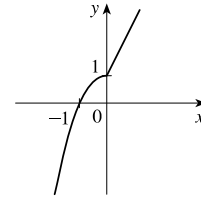


- (h) Note that $y = 1 + x^{-1} = 1 + 1/x$. So first plot $y = 1/x$ and then shift it upward 1 unit.



6. (a) $f(-2) = 1 - (-2)^2 = -3$ and $f(1) = 2(1) + 1 = 3$

- (b) For $x \leq 0$ plot $f(x) = 1 - x^2$ and, on the same plane, for $x > 0$ plot the graph of $f(x) = 2x + 1$.



7. (a) $(f \circ g)(x) = f(g(x)) = f(2x - 3) = (2x - 3)^2 + 2(2x - 3) - 1 = 4x^2 - 12x + 9 + 4x - 6 - 1 = 4x^2 - 8x + 2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 = 2x^2 + 4x - 2 - 3 = 2x^2 + 4x - 5$

(c) $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(2x - 3)) = g(2(2x - 3) - 3) = g(4x - 9) = 2(4x - 9) - 3 = 8x - 18 - 3 = 8x - 21$

Test D Trigonometry

1. (a) $300^\circ = 300^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{300\pi}{180} = \frac{5\pi}{3}$

(b) $-18^\circ = -18^\circ \left(\frac{\pi}{180^\circ} \right) = -\frac{18\pi}{180} = -\frac{\pi}{10}$

2. (a) $\frac{5\pi}{6} = \frac{5\pi}{6} \left(\frac{180}{\pi} \right)^\circ = 150^\circ$

(b) $2 = 2 \left(\frac{180}{\pi} \right)^\circ = \left(\frac{360}{\pi} \right)^\circ \approx 114.6^\circ$

3. We will use the arc length formula, $s = r\theta$, where s is arc length, r is the radius of the circle, and θ is the measure of the central angle in radians. First, note that $30^\circ = 30^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{\pi}{6}$. So $s = (12) \left(\frac{\pi}{6} \right) = 2\pi$ cm.

4. (a) $\tan(\pi/3) = \sqrt{3}$ [You can read the value from a right triangle with sides 1, 2, and $\sqrt{3}$.]

- (b) Note that $7\pi/6$ can be thought of as an angle in the third quadrant with reference angle $\pi/6$. Thus, $\sin(7\pi/6) = -\frac{1}{2}$, since the sine function is negative in the third quadrant.

- (c) Note that $5\pi/3$ can be thought of as an angle in the fourth quadrant with reference angle $\pi/3$. Thus,

$$\sec(5\pi/3) = \frac{1}{\cos(5\pi/3)} = \frac{1}{1/2} = 2, \text{ since the cosine function is positive in the fourth quadrant.}$$

8 □ DIAGNOSTIC TESTS

5. $\sin \theta = a/24 \Rightarrow a = 24 \sin \theta$ and $\cos \theta = b/24 \Rightarrow b = 24 \cos \theta$

6. $\sin x = \frac{1}{3}$ and $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos x = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$. Also, $\cos y = \frac{4}{5} \Rightarrow \sin y = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$.

So, using the sum identity for the sine, we have

$$\sin(x + y) = \sin x \cos y + \cos x \sin y = \frac{1}{3} \cdot \frac{4}{5} + \frac{2\sqrt{2}}{3} \cdot \frac{3}{5} = \frac{4 + 6\sqrt{2}}{15} = \frac{1}{15}(4 + 6\sqrt{2})$$

7. (a) $\tan \theta \sin \theta + \cos \theta = \frac{\sin \theta}{\cos \theta} \sin \theta + \cos \theta = \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$

(b) $\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \sin x / (\cos x)}{\sec^2 x} = 2 \frac{\sin x}{\cos x} \cos^2 x = 2 \sin x \cos x = \sin 2x$

8. $\sin 2x = \sin x \Leftrightarrow 2 \sin x \cos x = \sin x \Leftrightarrow 2 \sin x \cos x - \sin x = 0 \Leftrightarrow \sin x (2 \cos x - 1) = 0 \Leftrightarrow$
 $\sin x = 0$ or $\cos x = \frac{1}{2} \Rightarrow x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi.$

9. We first graph $y = \sin 2x$ (by compressing the graph of $\sin x$ by a factor of 2) and then shift it upward 1 unit.

