

Ex: $ty' = \frac{\sin t}{t} - 2y$, $t > 0$

Step 0: Standard form

$$y' + \frac{2}{t}y = \frac{\sin t}{t^2}$$

Step 1: Mult by μ s.t exact derivative on LHS

$$\underbrace{\mu y' + \frac{2}{t}\mu y}_{(\mu y)'} = \mu \frac{\sin t}{t^2}$$

Choose $\mu > 0$

$$\mu y'' + \mu' y \Rightarrow \mu' = \frac{2}{t}\mu \Rightarrow \int \frac{d\mu}{\mu} = \int \frac{2}{t} dt$$

$$\Rightarrow \ln \mu = 2 \ln t$$

\uparrow \uparrow
 $\mu > 0$ $t > 0$

$$\Rightarrow \mu = e^{2 \ln t} = e^{\ln t^2} = t^2$$

$\boxed{\mu = t^2}$

Step 2: Integrate

$$(\mu y)' = \mu \frac{\sin t}{t^2}$$

$$\Rightarrow (t^2 y)' = \sin t$$

$$\Rightarrow t^2 y = \int \sin t = -\cos t + C$$

Step 3: Solve for y

$$\boxed{y(t) = \frac{-\cos t}{t^2} + \frac{C}{t^2}}$$