Math 1522 - Exam 1 Review - Fall 2024

Exam 1 covers §6.1-6.4, 6.6-6.8 (unless told otherwise by your instructor)

Topics:

1. Inverse functions: *defining properties, graphs, derivatives*

2. Exponential, Logarithmic, Inverse Trigonometric, and Hyperbolic functions: *graphs, domains, ranges, values at particular x's, whether even or odd, limiting values, derivatives, logarithmic differentiation*

3. Integration: *know basic integrals and methods from Calc I (e.g. substitution), and the new integrals:* $\int e^x\,dx$, $\int \frac{1}{x}\,dx$, $\int \frac{1}{\sqrt{1-x^2}}\,dx$, $\int \frac{1}{1+x^2}\,dx$, $\int \frac{1}{x\sqrt{x^2-1}}\,dx$

4. L'Hopital's Rule: *compute limits with or without L'Hopital's rule, as appropriate (indeterminate forms* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$*), compute limits involving other indeterminate forms:* $0\cdot\infty$, $\infty-\infty$, 0^0, ∞^0, 1^∞

Sample Problems:

1. Sketch graphs of exponentials, logarithms, inverse trig functions, and their translations (e.g. HW1 problems 17, 29 and HW2 problem 17)

2. Find the equation for the tangent line to $f^{-1}(x)$ at $x=a$:
   (a) $f(x)=3+x^2+\tan(\pi x/2), \quad a=3$
   (b) $f(x)=\int_3^x \sqrt{1+t^3}\,dt, \quad a=0$

3. Evaluate the limits:
   (a) $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$
   (b) $\lim_{x\to 0^-}\tan^{-1}\left(\frac{\pi}{x}\right)$
   (c) $\lim_{x\to\infty}\frac{e^x+e^{-x}}{e^x-e^{-x}}$
   (d) $\lim_{x\to\infty}\frac{1+2^x}{1-2^x}$
   (e) $\lim_{x\to\infty}\left(1+\frac{r}{x}\right)^x$
   (f) $\lim_{x\to 0^+}x^x$

4. Find the derivatives of the following functions:
   (a) $f(x)=x^2\ln(x^3+1)$
   (b) $f(x)=e^{x^2}\tan^{-1}(\sqrt{x})$
   (c) $f(x)=\ln(\cosh(x))$
   (d) $f(x)=x2^x$
   (e) $f(x)=\frac{x^2\sqrt{x+2}}{(3x^2-1)^3}$
   (f) $f(x)=(\sqrt{x})^x$

   Hint: parts (d), (e), and (f) above can be done by logarithmic differentiation.

5. Evaluate the integrals:
   (a) $\int\sqrt{2r-1}\,dr$
   (b) $\int\frac{1}{2-3s}\,ds$
   (c) $\int\frac{e^{5x}}{e^{2x}}\,dx$
   (d) $\int\frac{r^3}{\sqrt{4+r^2}}\,dr$
   (e) $\int\frac{\cos(1/r)}{r^2}\,dr$
   (f) $\int x\sin(3x^2)\,dx$
   (g) $\int\frac{1}{x\sqrt{x^2-4}}\,dx$
   (h) $\int\frac{1}{x^2+5}\,dx$
   (i) $\int\tan x\,dx$

   Hint: for part (i) above, write $\tan x$ *as* $\frac{\sin x}{\cos x}$ *and then use substitution.*

6. Evaluate the definite integrals:
   (a) $\int_0^t(t-s)^2\,ds$
   (b) $\int_0^4\frac{1}{16+t^2}\,dt$
   (c) $\int_2^5\frac{1}{1+2r}\,dr$
   (d) $\int_0^1\frac{e^x}{1+e^{2x}}\,dx$
   (e) $\int_1^2\frac{(x+1)^2}{x}\,dx$
   (f) $\int_1^2\frac{x}{(x+1)^2}\,dx$

7. If a water wave with length L moves with velocity v in a body of water with depth d, then

$$v = \sqrt{\frac{gL}{2\pi}} \tanh\left(\frac{2\pi d}{L}\right)$$

where g is acceleration due to gravity. Explain why the approximation

$$v \approx \sqrt{\frac{gL}{2\pi}}$$

is appropriate in deep water.

8. §6.7: 62a (terminal velocity)