Math 1522 - Exam 2 Review - Fall 2024

Exam 2 covers §7.1-7.4, 7.7-7.8 (unless told otherwise by your instructor)

Topics:

1. **Methods of Integration**: Substitution, integration by parts, trigonometric integrals, trigonometric substitution, partial fractions.
2. **Approximation of Definite Integrals**: Riemann sum approximations L_n, R_n, M_n (using left end points, right end points, or mid points); trapezoid rule T_n.
3. **Improper Integrals**: Recognize improper integrals with unbounded domains or unbounded integrands. Rewrite as a limit or a sum of limits as appropriate. Evaluate the limits to determine whether the improper integral converges or diverges.

Sample Problems:

1. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{a}{x^2 - bx}\,dx$

   (b) $\displaystyle\int \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\,dx$

   (c) $\displaystyle\int \sin^2(3x)\,dx$

   (d) $\displaystyle\int \frac{1}{9 - x^2}\,dx$

   (e) $\displaystyle\int \frac{\sqrt{x^2 - 9}}{x}\,dx$

   (f) $\displaystyle\int \frac{1}{2t^2 + 3t + 1}\,dt$

   (g) $\displaystyle\int \frac{x^3 + 4}{x^2 + 4}\,dx$

   (h) $\displaystyle\int \frac{1}{16 + t^2}\,dt$

   (i) $\displaystyle\int \frac{1}{1 + 2r}\,dr$

   (j) $\displaystyle\int \sin^3\theta \cos^2\theta\,d\theta$

   (k) $\displaystyle\int x^5 \ln x\,dx$

   (l) $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{x^3}{\sqrt{x^2 + 4}}\,dx$

   (m) $\displaystyle\int_0^a x^2\sqrt{a^2 - x^2}\,dx$

   (n) $\displaystyle\int_0^t (t - s)^2\,ds$

   (o) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x}\,dx$

   (p) $\displaystyle\int_1^2 \frac{(x+1)^2}{x}\,dx$

   (q) $\displaystyle\int_1^2 \frac{x}{(x+1)^2}\,dx$

   (r) $\displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}}\,dx$

2. (a) Find the approximation T_4 (trapezoid rule using 4 subintervals) of the integral $I = \displaystyle\int_0^1 e^x\,dx$.

   (b) Sketch a graph of $f(x) = e^x$ and the area given by the approximation T_4 from part (a). Use your sketch to determine whether T_4 is an overestimate or underestimate.

3. For which values of p does $\displaystyle\int_a^\infty \frac{1}{x^p}\,dx$ converge? For which values of p does the integral diverge?

4. Determine if the following integrals converge or diverge. If they converge, compute their value. Use correct notation throughout.

   (a) $\displaystyle\int_{-\infty}^{\infty} \cos(\pi t)\,dt$

   (b) $\displaystyle\int_0^1 \frac{x - 1}{\sqrt{x}}\,dx$

   (c) $\displaystyle\int_0^3 \ln x\,dx$

   (d) $\displaystyle\int_3^\infty \ln x\,dx$

   (e) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6}\,dx$

   (f) $\displaystyle\int_3^6 \frac{1}{\sqrt{x - 3}}\,dx$

   (g) $\displaystyle\int_0^1 \frac{2}{1 - 3s}\,ds$

   (h) $\displaystyle\int_{-\infty}^{\infty} ye^{-3y}\,dy$

   (i) $\displaystyle\int_0^\infty e^{-as}\,ds,\ a > 0$

5. §7.8 Problem 74 (average speed of molecules in ideal gas)
   Hint: begin by setting $a = M/(2RT)$, thus simplifying the integrand.

6. Chapter 7 Review, page 593, Problem 87 (evaluate $\int_0^\infty f'(x)\,dx$ if $\lim_{x\to\infty} f(x) = 0$)