HOMEWORK DAY 4 – *Derivatives of* $\ln x, \log_b(x), b^x$ *§6.4*

1. §6.4: 2.

2. §6.4: 4.

3. §6.4: 5.

4. §6.4: 6.

5. §6.4: 7.

6. §6.4: 11.

7. §6.4: 13.

8. §6.4: 17.

9. §6.4: 47 (logarithmic differentiation)

10. §6.4: 52 (logarithmic differentiation)

11. Consider the function $f(x) = x \ln x$

   (a) State the domain of f.

   (b) Find all intercepts.

   (c) Find where f is increasing/decreasing.

   (d) Find where f is concave up/down.

   (e) Find $\lim_{x \to \infty} f(x)$.

   (f) Using a table of values, estimate $\lim_{x \to 0^+} f(x)$. (Include your table at the margin of this page.)

   (g) Use the above information to sketch a graph of $f(x)$

12. §6.4: 76.

13. §6.4: 81.

14. §6.4: 84.

15. §6.4: 86.

16. *Values of inverse trig functions.* Find the following values (without using a calculator).

    (a) $\sin^{-1}\left(\frac{1}{2}\right)$.

    Answer : $\sin^{-1}\left(\frac{1}{2}\right) = \alpha$ where $\sin(\alpha) = \frac{1}{2}$, $\alpha \in [-\pi/2, \pi/2]$ (range of $\sin^{-1}$). So $\alpha = \pi/6$.

    (b) $\cos^{-1}\left(\frac{1}{2}\right)$

    (c) $\sin^{-1}(1)$

    (d) $\cos^{-1}(-1)$

    (e) $\tan^{-1}(-\sqrt{3})$

    (f) $\sec^{-1}(2)$

    (g) $\cos^{-1}(\sqrt{3}/2)$

17. *Graphs of inverse trig functions.* Sketch graphs of the following functions on two side-by-side plots (make a separate plot for each function).

(a) $y = \sin x, -\pi/2 \le x \le \pi/2$ and $y = \sin^{-1} x$.

(b) $y = \cos x, 0 \le x \le \pi$ and $y = \cos^{-1} x$

(c) $y = \tan x, -\pi/2 < x < \pi/2$ and $y = \tan^{-1} x$

18. By drawing the appropriate triangle, find

    (a) $\cos(\sin^{-1} x)$ (Hint: draw a right triangle with angle $\alpha = \sin^{-1} x$, that is $\sin \alpha = \frac{x}{1} = \frac{\text{opp}}{\text{hyp}}$)

    (b) $\tan(\sin^{-1} x)$

19. *Derivatives of inverse trig functions.* Find the derivatives of the following functions. Simplify if possible.

    (a) §6.6: 22.

    (b) §6.6: 23.

    (c) §6.6: 24.

    (d) §6.6: 25.

    (e) §6.6: 27.

    (f) §6.6: 35.

20. *Limits of inverse trig functions.* Find the following limits

   (a) §6.6: 45.

   (b) §6.6: 46.

   (c) §6.6: 47.

   (d) §6.6: 48.

21. *Integrals.* Evaluate the following integrals

   (a) §6.6: 69. $\displaystyle\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}dx = \int e^u du = e^u + C = e^{\sin^{-1}x} + C$ where $u = \sin^{-1}x, du = \frac{1}{\sqrt{1-x^2}}dx$

(b) §6.6: 71.

(c) §6.6: 72.

(d) §6.6: 73.

(e) §6.6: 74.