HOMEWORK DAY 7 – *Hyperbolic functions §6.7*

1. Find the derivative of the following functions. Simplify if possible.

   (a) §6.7: 36

   (b) §6.7: 38

   (c) §6.7: 39

   (d) §6.7: 40

   (e) §6.7: 41

   (f) §6.7: 42

2. Prove that $\cosh^2(x) - \sinh^2(x) = 1$

3. Sketch the graph of $y = \cosh x$. In your sketch, indicate all the solutions to $\cosh x = 2$. Then solve $\cosh x = 2$ to find the exact values. (Hint: use the definition $\cosh x = (e^x + e^{-x})/2$.)

4. Use the definition $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ to answer the questions below.

(a) Is $\tanh(x)$ odd or even? Show your work. (Use the definition of an odd/even function.)

(b) Find $\lim_{x\to\infty} \tanh(x)$

(c) Find $\lim_{x\to-\infty} \tanh(x)$

(d) State all intercepts and all asymptotes of $y = \tanh(x)$:

(e) Show that $f(x) = \tanh(x)$ is always increasing.

(f) Use the information above to sketch a graph of $y = \tanh(x)$.

5. If a water wave with length L moves with velocity v in a body of water with depth d, then

$$v = \sqrt{\frac{gL}{2\pi}} \tanh\left(\frac{2\pi d}{L}\right)$$

where g is acceleration due to gravity. Explain why the approximation

$$v \approx \sqrt{\frac{gL}{2\pi}}$$

is appropriate in deep water.

HOMEWORK DAY 8 – *L'Hôpital's rule §6.8*

6. §6.8: 32. $\lim_{x\to\infty} \frac{(\ln x)^2}{x} \overset{(\frac{\infty}{\infty})}{\underset{LH}{=}} \lim_{x\to\infty} \frac{2(\ln x)/x}{1} = \lim_{x\to\infty} \frac{2\ln x}{x} \overset{(\frac{\infty}{\infty})}{\underset{LH}{=}} \lim_{x\to\infty} \frac{2/x}{1} = 0$

7. §6.8: 10.

8. §6.8: 11.

9. §6.8: 13.

10. §6.8: 14.

11. §6.8: 15.

12. §6.8: 24.

13. §6.8: 25.

14. A drug response curve describes the level of medication in the bloodstream after a drug is administered. A surge function $S(t) = At^p e^{-kt}$ is often used to model the response curve, reflecting an initial surge in the drug level and then a more gradual decline. The parameters $A, k > 0$ are positive and depend on the particular drug.

    Consider the drug response curve $S(t) = At^2 e^{-kt}$, $\quad t \geq 0$.

    (a) Find all intercepts.

(b) Find $\lim_{t\to\infty} S(t) =$

(c) Find all local minimums and maximums of $S(t)$ for $t \geq 0$. Justify your answer.

(d) Use your results to sketch a graph of the function.

(e) What is the maximal drug response and when does it occur? Label that point in your graph.

15. Let $f(x) = \dfrac{\sin x}{x}$.

(a) Determine whether f is even or odd.

(b) State the domain and find all intercepts.

(c) Find $\lim_{x\to\infty} f(x) =$

(d) Find $\lim_{x\to 0} f(x) =$

(e) Use the above to sketch a clearly labeled graph of f for $x \in [-5\pi, 5\pi]$.

16. Find the following limits, if they exist.

(a) $\lim_{x \to 0^+} x \ln x$

(b) $\lim_{x \to -\infty} xe^x$

(c) $\lim_{t \to -\infty} At^2 e^{-kt}$, where $A, k > 0$

(d) $\lim_{x \to \infty} \dfrac{\ln(x)}{\sqrt{x}}$

(e) $\lim_{x \to 1} \dfrac{\ln(x)}{\sqrt{x}}$

(f) $\lim_{x \to 0^+} \dfrac{\ln(x)}{\sqrt{x}}$

(g) $\lim_{x \to \infty} \Big(\ln(x) - \ln(x+1) \Big)$

(h) $\lim_{n \to \infty} \left(1 + \dfrac{r}{n}\right)^n$, $r \in \mathbb{R}$