HOMEWORK DAY 21 – *Sequences §11.1*

1. (a) What is a sequence?

   (b) What does it mean to say that $\lim_{n\to\infty} a_n = 3$?

   (c) What does it mean to say that $\lim_{n\to\infty} a_n = \infty$?

2. What are the following limits? Illustrate with a graph.

   (a) $\lim_{n\to\infty} a_n$, where $a_n = \cos(n\pi)$

   (b) $\lim_{n\to\infty} a_n$, where $a_n = \cos((n + 1/2)\pi)$

3. List the first five terms of the sequence, $a_n = \frac{1}{3^{n+1}}$, $n = 1, 2, \ldots$

4. Find a formula for the general term a_n of the sequence, assuming the pattern for the first few terms continues

   (a) $\{2, 4, 6, 8, \ldots\}$

   (b) $\{1, 3, 5, 7, \ldots\}$

   (c) $\{1, -1, 1, -1, \ldots\}$

   (d) $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$

   (e) $\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, -\frac{1}{256}, \ldots\}$

5. §11.1: 25

6. Determine the limit of the following sequences or explain why it does not exist. Show all necessary work. Use the fact that if $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} f(n) = L$ (where n is restricted to integers).

   (a) $a_n = \dfrac{1}{2^n}$

(b) $a_n = \dfrac{n-2}{n^2}$

(c) $a_n = \dfrac{\sin(n)}{n}$

(d) $a_n = \dfrac{n}{n+1}$

(e) $a_n = (-1)^n \dfrac{n}{n+1}$

(f) $a_n = n^{1/n}$

7. Give an example showing that if $\lim_{n\to\infty} f(n) = L$ (where n is restricted to integers), it is not necessarily true that $\lim_{x\to\infty} f(x) = L$ (where x is a continuous real variable).

8. (a) What does it mean to state that $\displaystyle\sum_{k=1}^{\infty} a_k = 5$?

   (b) Assume $\displaystyle\sum_{k=1}^{\infty} a_k = 5$. What is $\displaystyle\lim_{k\to\infty} a_k$?

   What is $\displaystyle\lim_{n\to\infty} s_n$, where $s_n = \displaystyle\sum_{k=1}^{n} a_k$?

   (c) Find the value of the series $\displaystyle\sum_{k=1}^{\infty} a_k$ if its partial sums are given by $s_n = \dfrac{n^2-1}{4n^2+1}$.

   (d) Find $\displaystyle\sum_{n=1}^{\infty} \cos n$ or determine it does not converge.

   (e) Find $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)$ or determine it does not converge.

9. Fill in the blanks using either *may* or *must*.

   (a) A series $\sum_{n=1}^{\infty} a_n$ whose terms a_n tend to 0 as $n \to \infty$ __________ converge.

   (b) A series $\sum_{n=1}^{\infty} a_n$ that converges __________ have terms a_n that tend to 0 as $n \to \infty$.

   (c) If a series $\sum_{n=1}^{\infty} a_n$ diverges, then the terms a_n __________ not tend to 0 as $n \to 0$.

   (d) If a series diverges, then the Divergence Test ________ succeed in proving the divergence.

   (e) If $\sum_{n=10}^{\infty} a_n$ diverges, then $\sum_{n=1000}^{\infty} a_n$ __________ diverge.

10. Evaluate the following series or determine that they diverge (with very brief explanation).

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

(b) $\displaystyle\sum_{n=2}^{\infty} 0.4^n$

(c) $\displaystyle\sum_{n=-3}^{\infty} \frac{1}{(-5)^n}$

(d) $1 + 0.4 + 0.16 + 0.064 + \dots$

11. Evaluate the following series or determine that they diverge (with very brief explanation).

(a) §11.2: 32

(b) §11.2: 36

(c) §11.2: 38

(d) §11.2: 44

(e) $\displaystyle\sum_{n=3}^{\infty}(5^{-n} + 2 \cdot 3^{-n})$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{7^{n-1}}{(-9)^n}$

(g) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(h) $\sum_{n=1}^{\infty} (\tan^{-1}(n+2) - \tan^{-1} n)$