HOMEWORK DAY 23 – *Integral Test, p-series §11.3*

1. Use the Integral Test to determine whether the series is convergent or divergent. Carefully show all details.

   (a) §11.3: 5

   (b) §11.3: 9

(c) §11.3: 10

2. Determine whether the following series converge or diverge (with a brief, but complete, explanation).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$

(b) $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \dots$

(c) $\frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

(g) $1 + \dfrac{10}{4} + \dfrac{10}{9} + \dfrac{10}{16} + \dots$

(h) $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$

(i) $\sum_{n=100}^{\infty} \frac{4^n}{3^n + 3}$

(j) $\sum_{n=2}^{\infty} 2^n$

(k) $\sum_{n=1}^{\infty} e^{-n}$

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ converges or diverges, using the following 3 methods. Give all necessary details in each case.

   (a) Direct Comparison Test:

   (b) Limit Comparison Test:

   (c) Integral Test:

4. Determine whether the following series converge or diverge, using the method of your choice (with brief but complete explanation).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \ln n$

(d) $\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\dots$

(e) $\sum_{n=1}^{\infty}\frac{1+\cos n}{e^n}$

(f) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n^4+1}}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{1+n}}{2+n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \sin(\frac{1}{n})$

(i) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$

(j) $\sum_{n=1}^{\infty}\frac{\tan^{-1} n}{n^2+1}$

5. (a) What is an alternating series?

   (b) Under what conditions does an alternating series converge? Answer concisely.
   Note: In all problems below you need to make sure to check whether these conditions are satisfied if you are using the alternating series test.

   (c) If these conditions are satisfied, what can you say about the remainder $R_n = s - s_n$, where s is the sum of the infinite series and s_n is the nth partial sum?

6. Determine whether the series converges or diverges.

   (a) $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \dots$

(b) $\sum_{k=2}^{\infty} \frac{1}{\ln k}$

(c) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{\ln k}$

(d) $\sum_{k=1}^{\infty} (-1)^k \frac{3k-1}{2k+1}$

7. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$

(b) §11.5: 22

(c) §11.5: 24

(d) §11.5: 28

(e) §11.5: 34

8. §11.5: 37

9. §11.5: 38