HOMEWORK DAY 26 – *Ratio Test §11.6*

1. §11.6: 1

2. §11.6: 3

3. §11.6: 7

4. §11.6: 8

5. §11.6: 12

6. §11.6: 15

7. Determine whether the following series converge absolutely, converge conditionally, or diverge. State the tests that you used and give a brief explanation.

   For example: For problem 11.6:13 you could write "Converges absolutely by direct comparison test, since $|a_n| = |\cos(n\pi/3)/n!| \leq 1/n!$, and $\sum 1/n!$ converges by the ratio test since ... (show it), so $\sum |a_n|$ converges."

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{\sqrt{k+4}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(c) $\displaystyle\sum_{n=3}^{\infty} \frac{4^n}{5^n - 1}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

(e) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^3+1}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{100^n}{n!}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$

8. Which of the following are power series? Circle the ones that are.

(a) $\displaystyle\sum_{n=0}^{\infty}(3x)^n$ (b) $\displaystyle\sum_{n=0}^{\infty}\sqrt{x}^n$ (c) $\displaystyle\sum_{n=0}^{\infty}(x+2)^{2n}$

9. Find the radius of convergence and the interval of convegence of the power series

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n}$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$

(c) $\displaystyle\sum_{n=10}^{\infty} \frac{(x+1)^n}{n^3 3^n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^n}$

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{2^n(x-2)^n}{n^{1/3}}$

(f) $\sum_{n=100}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

10. Suppose $\sum_{n=1}^{\infty} a_n 3^n$ converges and $\sum_{n=1}^{\infty} a_n 6^n$ diverges. What can you say about

(a) the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$?

(b) the series $\sum_{n=1}^{\infty} a_n 2^n$?

(c) the series $\sum_{n=1}^{\infty} a_n$?

(d) the series $\sum_{n=1}^{\infty} a_n 8^n$?

(e) the series $\sum_{n=1}^{\infty} a_n (-3)^n$?

(f) the interval of convergence of the power series $\sum_{n=1}^{\infty} a_n (x-a)^n$?

11. §11.9: 1

12. §11.9: 2

13. Find power series representations for the following functions about $x = a$ by manipulating known series. State their radius and interval of convergence.

    (a) $f(x) = \dfrac{1}{1+x}$, $a = 0$

(b) $f(x) = \dfrac{x}{1+x^2}$, $a = 0$

(c) $f(x) = \dfrac{1}{1+x}$, $a = 3$

(d) $f(x) = \tan^{-1}(x)$, $a = 0$

(e) $f(x) = \tan^{-1}(x^4)$, $a = 0$

(f) $f(x) = \dfrac{2}{3-x}$, $a = 0$

14. (a) Find a power series representation of $f(x) = \dfrac{1}{(1-x)^2}$ about $a = 0$ (hint: use differentiation of the geometric series).

(b) Use your result above to find a power series representation of $f(x) = \dfrac{x}{(2-x)^2}$ about $a = 0$.

15. *Application: approximating functions.*

(a) Use the geometric series to find a power series representation for $\ln(1+x)$ about $x = 0$.

(b) Find a power series representation for $\ln(1+x^2)$ about $x = 0$.

(c) Use the resulting series to estimate the value of $\ln(1.01)$ to within 10^{-7} (roughly single machine precision).