HOMEWORK DAY 38 – *Direction fields. Euler's Method §9.2*

1. Sketch the direction fields for the following differential equations. Then sketch a few integral curves (solution curves) tangent to the direction field.

   (a) $\dfrac{dy}{dt} = t$

   (b) $\dfrac{dy}{dt} = y$

   (c) $\dfrac{dy}{dt} = y^2$

Instructions. In the next two problems you need to implement Euler's method by hand. Report your answer in rows

$$\begin{array}{ll} t_0 = \dots & y_0 = \dots \\ t_1 = \dots & y_1 = \dots \\ \vdots & \\ t_n = \dots & y_n = \dots \end{array}$$

2. Consider the initial value problem

$$\frac{dy}{dt} = t^2 y \ , \qquad y(1) = 1 \ .$$

(a) Use Euler's method (by hand) to approximate the solution $y(t)$ at $t = 2$ using $\Delta t = 1$.

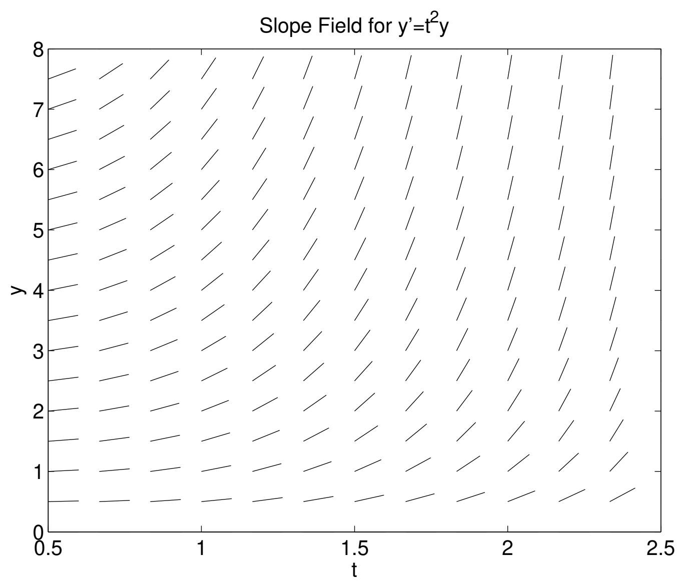
(b) Repeat using $\Delta t = 1/2$

(c) Repeat using $\Delta t = 1/4$ (use a calculator to approximate the answer at $t = 2$).

(d) The direction field of $\dfrac{dy}{dt} = t^2 y$ is shown below. In this figure, add a sketch of

∘ the exact solution $y(t) = e^{\frac{1}{3}(t^3-1)}$ to the given initial value problem, for $t \in [1, 2]$ (it may leave the given window)

∘ the numerical approximation using Euler's method with $\Delta t = 1/4$ that you found above (indicate the solution obtained at each step, and a line showing how it is obtained)



3. Consider the initial value problem

$$\frac{dy}{dx} = y - 2x \ , \quad y(1) = 0$$

(a) Use Euler's method (by hand) to approximate the solution $y(x)$ at $x = 3$ using using $\Delta x = 1$.

(b) Repeat the above using step size $\Delta x = 1/2$.

4. Solve the differential equation or initial value problem.

   (a) $\dfrac{dy}{dx} = 3x^2y^2$

   (b) $\dfrac{dy}{dx} = x\sqrt{y}$

(c) $xy' = 2y + 1, \; y(1) = 0$

(d) $\dfrac{dy}{dx} = 2x(y^2 + 1)$

(e) $\dfrac{dp}{dt} = t^2p - p + t^2 - 1$

5. Which of the equations in §9.5:1-4 is linear?

6. Find a positive solution $\mu(x)$ to each of the following equations (use any method you like).

   (a) $\mu' = -\mu$

   (b) $\mu' = 2\mu$

   (c) $\mu' = \dfrac{\mu}{x}$

   (d) $\mu' = \dfrac{4\mu}{x}$

7. Solve the following differential equations using the method of integrating factors.

(a) $4x^3y + x^4y' = \sin^3 x$

(b) $t^2\dfrac{dy}{dt} + 3ty = \sqrt{1+t^2}$, $t > 0$

8. Solve the initial value problem $y' = y - 2x$, $y(1) = 0$.

9. Show that the following terms are exact derivatives (they are of the form $\frac{d}{dx}[g(x)y]$ for some g).

(a) $xy' + y$

(b) $t^3 \frac{dy}{dt} + 3t^2 y$

10. Use your answer in (a) above to solve $xy' + y = \sqrt{x}$