Math 2531 - Exam 1 Review - Fall 2024

Exam 1 covers §12.1-12.6, 13.1 (unless told otherwise by your instructor)

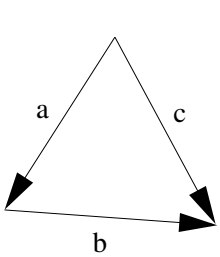
Topics:

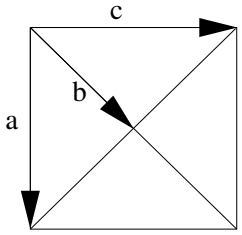
1. **Vectors**: vector addition and scalar multiplication (geometrically and componentwise), magnitude, standard basis vectors, unit vectors, resultant forces
2. **Dot Product and Cross Product**: Compute dot and cross product of two vectors, magnitude and direction of cross product. What does $\mathbf{a} \cdot \mathbf{b} = 0$ mean? $\mathbf{a} \cdot \mathbf{b} > 0$? $\mathbf{a} \cdot \mathbf{b} < 0$? $\mathbf{a} \times \mathbf{b} = \mathbf{0}$? What is $\mathbf{a} \cdot \mathbf{a}$? What is $\mathbf{a} \times \mathbf{a}$? Know relation between dot/cross product of two vectors and the enclosed angle, find vector and scalar projections of $\mathbf{b}$ onto $\mathbf{a}$, compute areas of triangles and parallelograms, find components of force in one direction, find vector normal to plane, find distances to lines/planes, etc.
3. **Lines and Planes**: Find equations for lines/planes. Find intersections of lines and lines, lines and planes, planes and planes. Find angle between lines and lines, lines and planes, planes and planes. Determine when lines are parallel, skew, intersect. Use projections to find distance from point to plane/line.
4. **Surfaces in 3D**: Sketch planes, cylinders, quadratic surfaces (use traces)
5. **Vector Functions**: Sketch elementary curves $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ (circles, helices, ellipses, lines, curves which can be rewritten in nonparametric form). Determine where two curves intersect. Determine where a curve and surface intersect. Given two particle trajectories, determine if and where the particles collide.

Sample Problems:

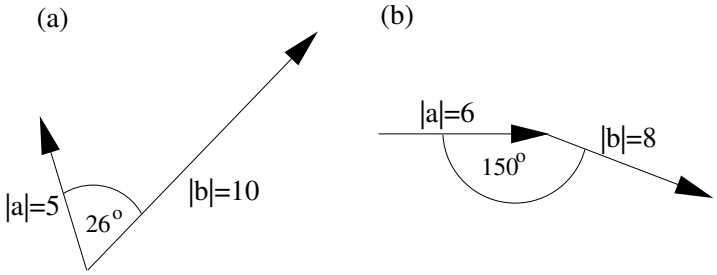
Vectors, Dot and Cross Products

1. Which of the following are meaningful? §12.3: 1, §12.4: 13
2. Find cross product using properties. §12.4: 9-12
3. Given a quadrilateral ABCD, let $\mathbf{a}_1 = \overrightarrow{AB}$, $\mathbf{a}_2 = \overrightarrow{BC}$, $\mathbf{a}_3 = \overrightarrow{CD}$, $\mathbf{a}_4 = \overrightarrow{DA}$.
   Graph the vector $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$. Find $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$.
4. §12.2: 5
5. Determine whether two vectors are parallel or perpendicular to each other. If not, is the enclosed angle acute or obtuse?
6. The following figures shows an equilateral triangle and a square, where $|\mathbf{a}| = 1$. Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c}$, and $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c} \times \mathbf{b}$ (use only the length of the vectors and the enclosed angles).





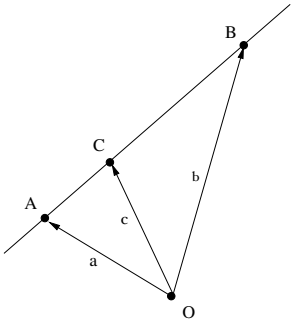
7. If $\mathbf{a}$ and $\mathbf{b}$ are the vectors shown in the figure, find $\mathbf{a}\cdot\mathbf{b}$ and $|\mathbf{a}\times\mathbf{b}|$. Determine whether $\mathbf{a}\times\mathbf{b}$ points into or out of the page.



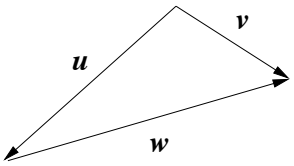
8. Chapter 12 Review, True-False Quiz: 1-22

9. Describe two methods to determine whether a triangle ABC is right-angled.

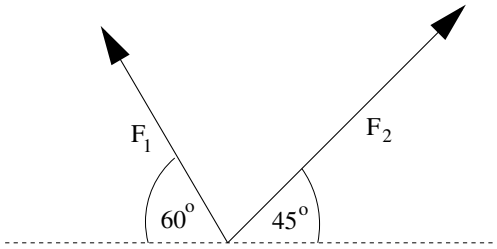
10. Let C be the point on the line segment AB that is twice as far from B as it is from A. Express $\mathbf{c}=\overrightarrow{OC}$ in terms of $\mathbf{a}=\overrightarrow{OA}$ and $\mathbf{b}=\overrightarrow{OB}$. See figure on the right.



11. Express $\mathbf{w}$ in terms of the vectors $\mathbf{u}$ and $\mathbf{v}$ in the figure below.



12. Let $\mathbf{F}_1$, $\mathbf{F}_2$ be as shown in the figure on the right, where $|\mathbf{F}_1|=3$ and $|\mathbf{F}_2|=4$.



   (a) Sketch the resultant force $\mathbf{F}$ in the figure.

   (b) Find the resultant force $\mathbf{F}$ and it's magnitude and elevation angle.

   Answer: $\mathbf{F}=\langle 2\sqrt{2}-\frac{3}{2}, 2\sqrt{2}+\frac{3\sqrt{3}}{2}\rangle$, $|\mathbf{F}|=\sqrt{25+6(\sqrt{6}-\sqrt{2})}$, $\theta=\tan^{-1}\left(\frac{4\sqrt{2}+3\sqrt{3}}{4\sqrt{2}-3}\right)\approx 76.24^o$

13. Use perpendicular component to find heights and areas of triangles and parallelograms and volumes of parallelepipeds: §12.4: 27, 29, 33

14. Draw any two vectors $\mathbf{a}$ and $\mathbf{b}$ of your choice. Then indicate the length $|\mathbf{a}\cdot\mathbf{b}|/|\mathbf{a}|$ in your picture. Repeat for $|\mathbf{a}\cdot\mathbf{b}|/|\mathbf{b}|$. Repeat for $|\mathbf{a}\times\mathbf{b}|/|\mathbf{a}|$. Repeat for $|\mathbf{a}\times\mathbf{b}|/|\mathbf{b}|$.

15. Chapter 12 Review, Exercises: 3, 4, 5, 6, 7, 11, 12, 13

16. Consider two vectors in the xy-plane, $\mathbf{a}=\langle a_1, a_2, 0\rangle$ and $\mathbf{b}=\langle b_1, b_2, 0\rangle$. Show that

$$|\mathbf{a}\times\mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2.$$

   (The same is true for any two vectors in space, but the algebra is slightly more involved – try it.)

Lines and Planes

17. Derive the vector and scalar equations of the straight line through (x_0, y_0, z_0) in the direction of the vector $\mathbf{v}=\langle a, b, c\rangle$. Make a sketch using vectors.

18. Derive the vector and scalar equations of the plane containing the point (x_0, y_0, z_0) and normal to the vector $\mathbf{n} = \langle a, b, c \rangle$ Make a sketch using vectors.

19. Write vector and parametric equations for line segments. §12.5: 17, 18

20. Find the distance from a point to a line. §12.5: 69, 70

21. Find the distance from a point to a plane. §12.5: 71, 72

22. Find the distance between parallel planes. §12.5: 73, 74

23. Determine whether two lines are parallel, identical, skew, or intersect at one point. §12.5: 19-22

24. Given two distinct lines that intersect, find the point of intersection. Find the plane spanned by the two lines. §12.5: 64

25. Find equations for the line of intersection of two planes, and find the angle between two intersecting planes. §12.5: 57, 58

26. Chapter 12 Review, Exercises: 15-27

Surfaces in 3D

27. Chapter 12 Review, Exercises: 1, 28-37

Vector Functions

28. §13.1: 1-4, 7-12, 25-30

29. Do particles' paths intersect? Do they collide? §13.1: 57, 58

30. Chapter 13 Review, Exercises: 6a