Math 2531 - Exam 2 Review - Fall 2024

Exam 2 covers §13.2-13.4, 14.1-14.7, HW 4-8 (unless told otherwise by your instructor)

Topics:

1. **Vector functions**: curves $\mathbf{r}(t)$ in $\mathbb{R}^2$ or $\mathbb{R}^3$, their derivatives, unit tangent vector $\mathbf{T}(t)$, principal unit normal vector $\mathbf{N}(t)$, arc length, curvature, velocity, speed, acceleration, tangential and normal components of acceleration

2. **Functions of several variables**: determine and sketch domain of $f(x,y)$ or $f(x,y,z)$, graph of a surface $z = f(x,y)$ in $\mathbb{R}^3$, level curves (contour map) in xy-plane for $f(x,y) =$ constant, level surfaces in $\mathbb{R}^3$ for $f(x,y,z) =$ constant.

3. **Limits and Continuity**: compute a limit $\lim_{(x,y)\to(a,b)} f(x,y)$ when it exists, use the two path test to determine that a limit doesn't exist, determine all points where a function is continuous.

4. **Derivatives and slopes**: partial derivatives, directional derivatives, gradient vector and its properties:
   - ∇f points in direction of max increase of f
   - $|\nabla f|$ gives max rate of increase of f
   - $\nabla f(x,y)$ is orthogonal to level curve of f at (x,y),
   - $\nabla f(x,y,z)$ is orthogonal to level surface of f at (x,y,z)
   - $\nabla f \cdot \mathbf{u}$ gives directional derivative of f in direction of unit vector $\mathbf{u}$

5. **Tangent planes and normals**: find equation of tangent plane to the surface $z = f(x,y)$ at a point, find equation of tangent plane to the surface $F(x,y,z) = c$ at a point, find a vector normal to surface $z = f(x,y)$ or $F(x,y,z) = c$ at a point.

6. **Linearization**: find the linear approximation $L(x,y)$ to $f(x,y)$ or $L(x,y,z)$ to $f(x,y,z)$ about some point, approximate Δf (change in f) using ΔL (change in L) or by using differentials.

7. **Chain Rule**: various cases (tree diagrams can be helpful), implicit differentiation.

8. **Maximum and Minimum Values**: find critical points and use 2nd derivative test to classify them (local max, local min, or saddle point), find absolute max/min of a continuous function f on a closed and bounded domain D (evaluate f at critical points in the interior of D and check f on the boundary of D).

Sample Problems:

1. Sketch curves and tangent vectors. §13.2: 3-8

2. Consider the curve $\mathbf{r}(t) = \langle \frac{t^3}{3}, t^2, 2t \rangle$. Find the unit tangent vector at the point P$(\frac{8}{3}, 4, 4)$.
   Answer: $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

3. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. §13.2: 25-28

4. Find a vector equation for the tangent line to the curve

$$x = 2\sin t, \quad y = 2\sin(2t), \quad z = 2\sin(3t)$$

   at the point $(1, \sqrt{3}, 2)$.
   Answer: $\mathbf{r}(t) = \langle 1, \sqrt{3}, 2 \rangle + t\langle \sqrt{3}, 2, 0 \rangle$

5. Chapter 13 Review, Exercises: 6ab, 8, 9, 11abd, 12, 13, 16, 17, 18, 19, 22
   Answer to 6ab: (a) $(\frac{15}{8}, 0, -\ln 2)$ *(b)* $x = 1 - 3t,\ y = 1 + 2t,\ z = t$
   Hint for 9: The angle of intersection of two curves is the angle between their tangent vectors at the point of intersection.

6. Chapter 14 Review, Concept Check: 1-8, 10-17

7. Chapter 14 Review, Exercises: 1-12, 13-37 odd, 42, 45-48, 51-56

8. Let $f(x, y) = x^2 + 3xy$

   (a) Write down the equation of the tangent plane to the graph of f at (1,1,4) in the form $z = L(x, y)$ where L is the linearization of f about (1,1,4).

   (b) Find a normal vector to the surface by writing the surface as a level surface of some $F(x, y, z)$. Use the normal vector to obtain the equation of the tangent plane at (1,1,4).

   (c) Confirm that the results in (a) and (b) are the same.

9. Show that the directional derivative of f in the direction of ∇f is $|\nabla f|$.