10.6 pf : Let $P(n)$ be the statement $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$, $\forall n \in \mathbb{N}$

$P(1)$ asserts $\frac{1}{1\cdot 2} = \frac{1}{1+1}$, which is true

Now, Assume $P(n)$ is true.

$P(n+1)$ asserts :

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)} + \frac{1}{(n+1)((n+1)+1)}$$

$$= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} \quad \dots \text{(by induction hypothesis)}$$

$$= \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$= \frac{\cancel{(n+1)}(n+1)}{\cancel{(n+1)}(n+2)} = \frac{(n+1)}{((n+1)+1)}$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true

& by induction we conclude $P(n)$ is true for all n. ☒

10.7 pf : Let $P(n)$ be the statement $1 + r + r^2 + \dots + r^n = (1-r^{n+1})/(1-r)$

for $\forall n \in \mathbb{N}$ and $r \neq 1$

$P(1)$ asserts $1 + r = (1-r^2)/(1-r)$

$$= [(1+r)\cancel{(1-r)}]/\cancel{(1-r)} = 1+r$$

$\therefore$ $P(1)$ is true

Now, assume $P(n)$ is true and consider $P(n+1)$, which asserts

$$1 + r + r^2 + \dots + r^n + r^{(n+1)}$$

$$= (1-r^{n+1})/(1-r) + r^{n+1} \quad \dots \text{(by induction hypothesis)}$$

$$= [(1-r^{n+1}) + r^{n+1}(1-r)]/(1-r)$$

$$= [1 - r^{n+1} + r^{n+1} - r^{n+2}]/(1-r)$$

$$= (1 - r^{(n+1)+1})/(1-r)$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true

& by induction we conclude $P(n)$ is true for all n. ☒