KEY TO 'EXTRA PRACTICE FOR TEST 2'

I LINEAR FUNCTIONS

1) $f(x) = 3x+5$

$g(x) = -2x+15$

(A) SOLVE $f(x)=0$

$3x+5=0$

$3x=-5$

$\boxed{x=-5/3}$

(B) SOLVE $f(x)=g(x)$.

$3x+5 = -2x+15$

$3x+2x=15-5$

$5x=10$

$\boxed{x=2}$

(C) SOLVE $f(x)<0$.

$3x+5<0$

$3x<-5$

$\boxed{x<-5/3}$ $\boxed{(-\infty,-5/3)}$

(D) SOLVE $f(x)\geq g(x)$.

$3x+5\geq -2x+15$

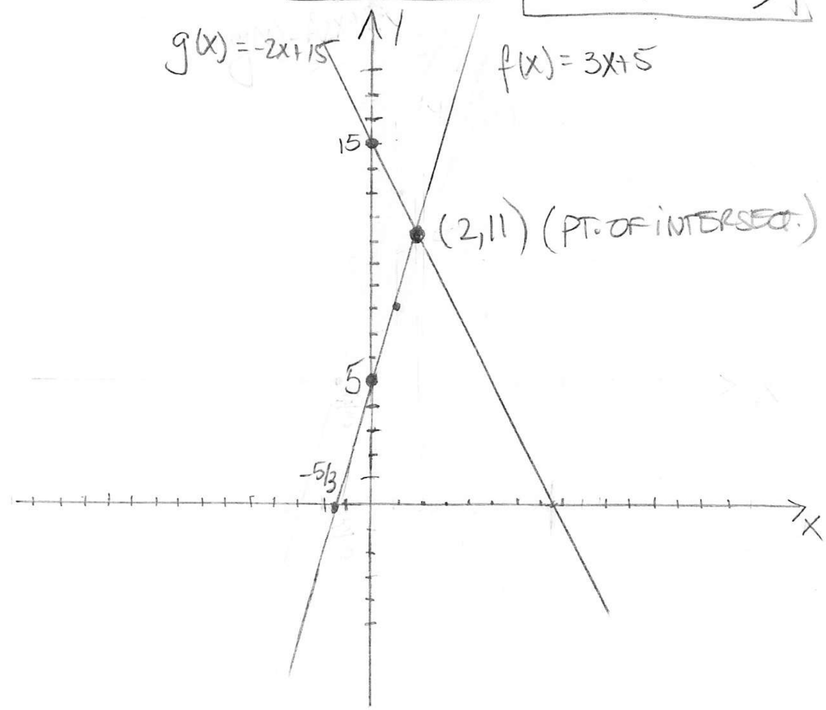
$3x+2x\geq 15-5$

$5x\geq 10$

$x\geq 2$

$[2,\infty)$

(E)



2. $C(x) = 0.25x+35$

(A) $C(40) = 0.25(40)+35$

$= \frac{1}{4}(40)+35$

$= 10+35$

$= 45$

THE COST IS \$45.

(B) $80 = 0.25x+35$

$0.25x = 45$

$\frac{1}{4}x = 45$

$x = 45(4)$

$\boxed{x = 180 \text{ MILES}}$

(C) $C(x)\leq 100$

$0.25x+35\leq 100$

$0.25x\leq 65$

$\frac{1}{4}x\leq 65$

$x\leq 4(65)$

$x\leq 260$

NO MORE THAN 260 MILES

3.

(A) $C(x) = 5+0.05x$

(B) $C(105) = 5+0.05(105)$

$= 5+5.25$

$= \$10.25$

$C(180) = 5+0.05(180)$

$= 5+9$

$= \$14$

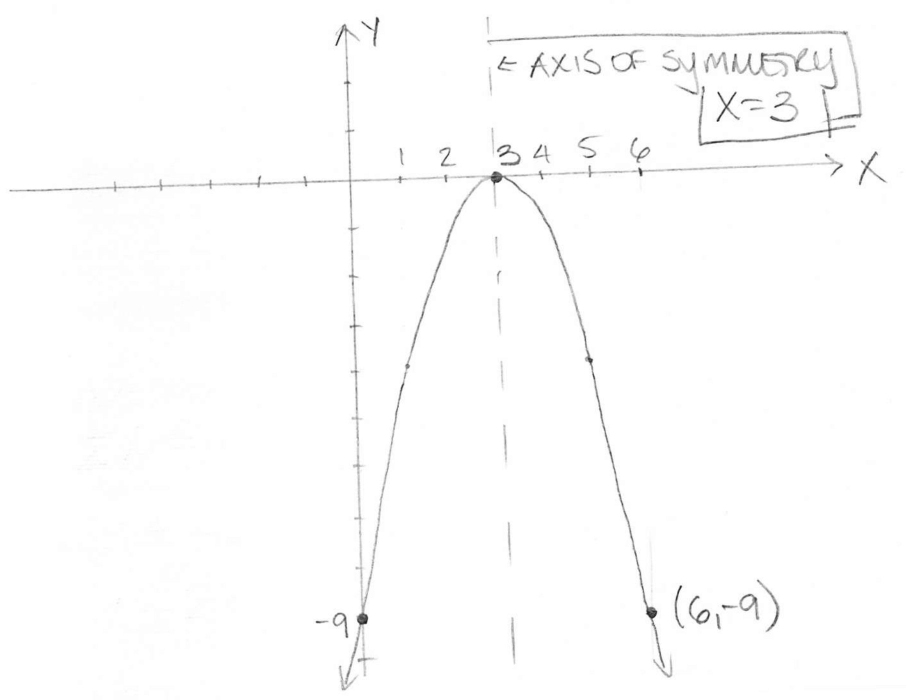
2. QUADRATIC FUNCTIONS

4. (A) ALL REAL #'S
$(-\infty, \infty)$
(B) PARABOLA
(C) UP
(D) MINIMUM
(E) AT THE VERTEX
(F) $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
(G) YOU CAN EITHER COMPL. THE SQUARE IN THE VARIABLE X OR USE $h = -\frac{b}{2a}$ & $K = f\left(-\frac{b}{2a}\right)$ & PLUG INTO $f(x) = a(x-h)^2 + K$
(H) $x = h = -\frac{b}{2a}$
(I) SET $Y=0$ & SOLVE THE RESULTING QUAD. EQ.
(J) FIND THE MIDPOINT B/WEEN THE X-INTERCEPTS.
(K) $f(0) = c \Rightarrow \boxed{(0, c)}$
(L) YES! IT IS A POLYNOMIAL OF DEGREE 2.

5. (A) $f(x) = -x^2 + 6x - 9$

- THE PARABOLA OPENS DOWN (B/C THE LEAD. COEFF. IS NEG.)
- X-INT $(Y=0)$
$-x^2 + 6x - 9 = 0$
$x^2 - 6x + 9 = 0$
$(x-3)^2 = 0$
$x = 3$
$\boxed{(3, 0)}$
- Y-INT $(X=0)$
$Y = -9 \quad \boxed{(0, -9)}$
- $h = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3$
$f(3) = -9 + 18 - 9 = 0$
$\boxed{(3, 0) - \text{V-TEX.}}$
- $f(x) = a(x-h)^2 + K$
$f(x) = -(x-3)^2 + 0$
$\boxed{f(x) = -(x-3)^2}$

- THIS PARABOLA HAS A MAXIMUM
THE MAXIMUM IS 0 AND IT OCCURS AT $X = 3$
- THE FN. IS INCREASING ON $(-\infty, 3)$
— " — DECREASING ON $(3, \infty)$
- DOMAIN: $(-\infty, \infty)$
RANGE: $(-\infty, 0]$
└▷ Y-COOR OF V-TEX
-



2.5. B) $K(x) = 2x^2 - 4x + 1$

- $a = 2 > 0$ opens up
- X-INT ($y=0$)

$2x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm \sqrt{8}}{4}$

$= \frac{4 \pm 2\sqrt{2}}{4} = 1 \pm \frac{\sqrt{2}}{2}$

$\left(1+\frac{\sqrt{2}}{2}, 0\right)$ & $\left(1-\frac{\sqrt{2}}{2}, 0\right)$

- Y-INT: $(0,1)$

- V-TEX: $(1,-1)$

$h = \frac{-b}{2a} = \frac{-(-4)}{2 \cdot 2} = 1$

$K(1) = 2 - 4 + 1 = -1$

- $K(x) = 2(x^2 - 2x \quad) + 1$

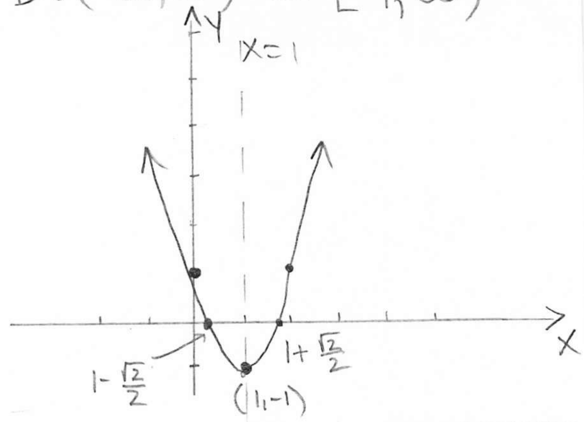
$= 2(x^2 - 2x + 1) - 2 + 1$

$= 2(x-1)^2 - 1$

$\boxed{K(x) = 2(x-1)^2 - 1}$ (h, K)

- AXIS OF SYMM: $x = 1$

- MINIMUM OF -1 AT $x = 1$
- INCR. ON $(1, \infty)$
DECR. ON $(-\infty, 1)$
- D: $(-\infty, \infty)$ R: $[-1, \infty)$



C) $g(x) = -2x^2 + 4$

- $a = -2 < 0$ DOWN
- X-INT ($y=0$)

$-2x^2 + 4 = 0$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$(\pm\sqrt{2}, 0)$

- Y-INT: $(0,4)$
- V-TEX $(0, 4)$

$h = -\frac{b}{2a} = \frac{0}{-4} = 0$

$g(0) = 4$

- AXIS OF SYMM $x = 0$
- $g(x) = -2(x-0)^2 + 4$

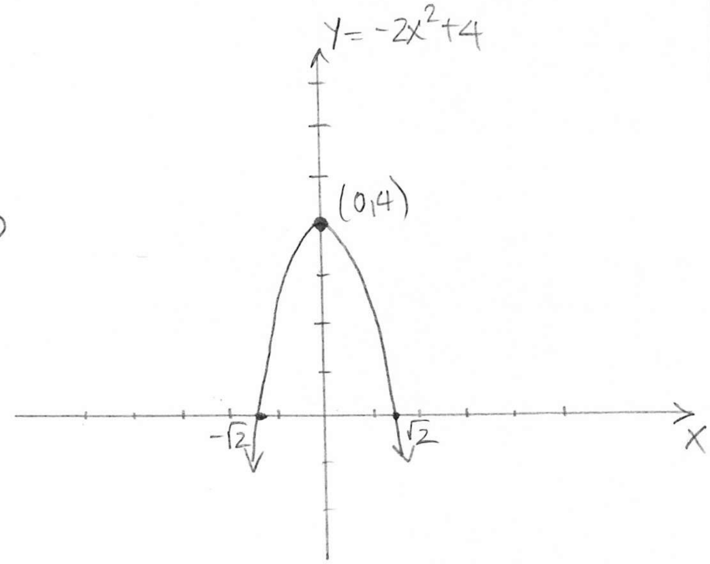
$g(x) = -2x^2 + 4$ ✓

- MAXIMUM OF 4 AT $x = 0$
- ↑ ON: $(-\infty, 0)$

↓ ON: $(0, \infty)$

- D: $(-\infty, \infty)$

R: $(-\infty, 4]$



D) $h(x) = 6 - x - x^2 = -x^2 - x + 6$

- $a = -1 < 0$ (DOWN)
- X-INT: $-x^2 - x + 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ OR $x = 2$

$\boxed{(-3,0) \ \& \ (2,0)}$

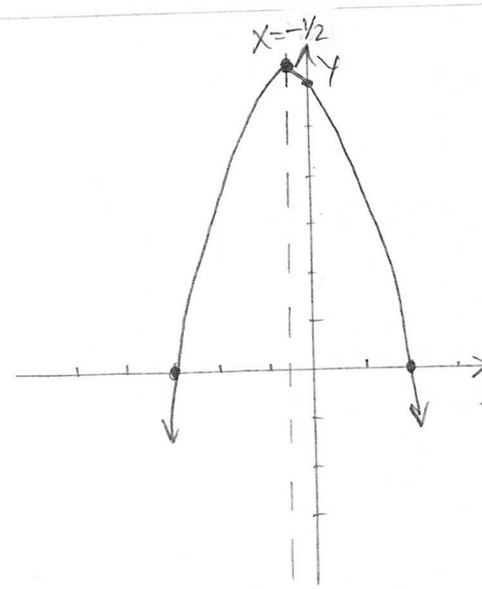
- Y-INT: $(0,6)$
- $\boxed{\text{V-TEX: } \left(-\tfrac{1}{2}, \tfrac{25}{4}\right)}$

- AXIS OF SYMM $x = -\frac{1}{2}$
- $h(x) = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$
- MAX OF $\frac{25}{4}$ AT $x = -\frac{1}{2}$
- ↑ ON: $(-\infty, -\frac{1}{2})$

↓ ON: $(-\frac{1}{2}, \infty)$

- D: $(-\infty, \infty)$

R: $(-\infty, \frac{25}{4}]$



$h = \frac{-b}{2a} = -\frac{-1}{2(-1)} = -\frac{1}{2}$; $h\left(-\frac{1}{2}\right) = -\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 6 = -\frac{1}{4} + \frac{1}{2} + 6 = \frac{-1+2+24}{4} = \frac{25}{4}$

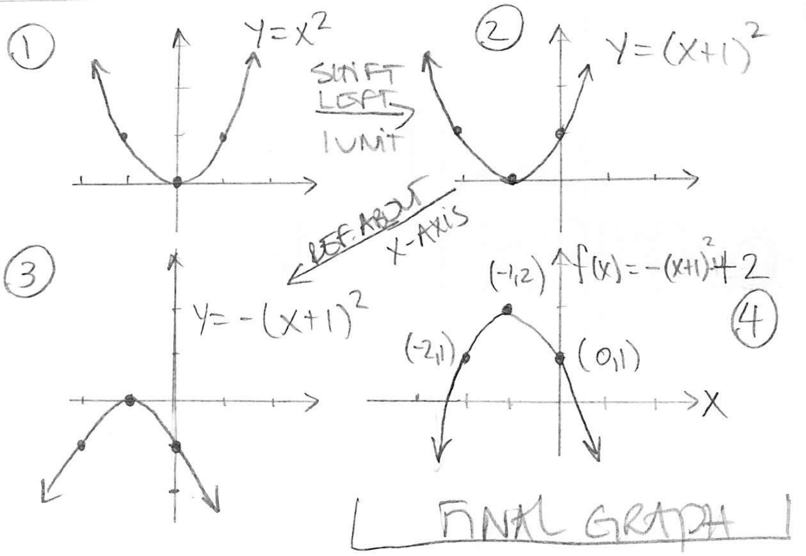
2. 6. $f(x) = -x^2 - 2x + 1$

$= -(x^2 + 2x \quad) + 1$

$= -(x^2 + 2x + 1) + 1 + 1$

$= -(x+1)^2 + 2$

$f(x) = \underbrace{-}_{\text{② REFLECT ABOUT THE X-AXIS}}\underbrace{(x+1)^2}_{\text{① SHIFT LEFT 1 UNIT}} \underbrace{+2}_{\text{③ SHIFT UP 2 UNITS}}$

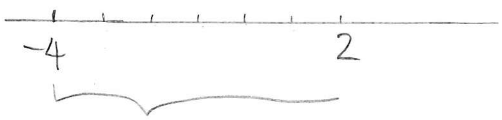


7. $f(x) = 3(x-2)(x+4)$

X-INT: $0 = 3(x-2)(x+4)$

$x = 2$ OR $x = -4$

$(2,0) \quad (-4,0)$



MIDPOINT

$h = \frac{-4+2}{2} = \frac{-2}{2} = -1$

$f(-1) = 3(-3)(3) = -27$

V-TEX $(-1, -27)$

8. $p = -\frac{1}{4}x + 40$

Ⓐ $R = x \cdot p = x\left(-\frac{1}{4}x + 40\right) \leftarrow$ ((#FLATS FIXED)(PRICE PER FLAT))

$R(x) = -\frac{1}{4}x^2 + 40x$

Ⓑ SEE GRAPH

$(0, 160)$

Ⓒ $R(40) = -\frac{1}{4}(40)^2 + 40(40) = -\frac{1600}{4} + 1600$

$\boxed{R(40) = \$1200}$

Ⓓ "WHAT IS THE X-COOR. OF THE V-TEX?"

$h = \frac{-b}{2a} = -\frac{40}{2(-1/4)} = 80$

HE SHOULD FIX 80 FLATS IN ORDER TO MAXIMIZE THE REV.

Ⓔ $p(80) = -\frac{1}{4}(80) + 40 = -20 + 40 = 20$

HE SHOULD CHARGE \$20 PER FLAT

Ⓕ $R(80) = -\frac{1}{4}(80)^2 + 40(80) = -\frac{1}{4}(6400) + 3200$

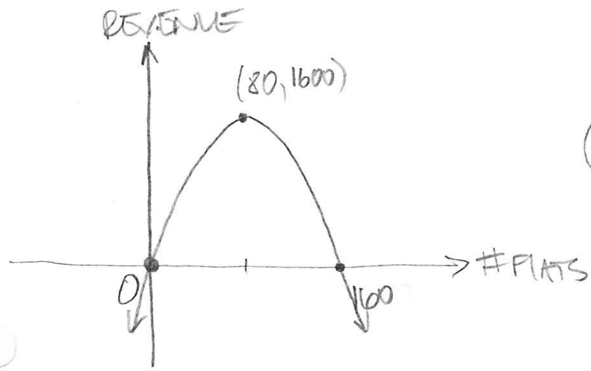
$\uparrow$

Y-COOR OF V-TEX

$= -1600 + 3200 = 1600$

MAX. REVENUE \$1600

Ⓑ



D: $(0, 160)$

(WE DON'T WANT NEGATIVE REVENUE)

9. $h(t) = -16t^2 + 32t + 48$

Ⓐ $h(0) = 48$

48 FT. ABOVE GROUND

Ⓑ $t = ?$ WHEN $h(t) = 48$

$-16t^2 + 32t + 48 = 48$

$-16t^2 + 32t = 0$

$-16t(t-2) = 0$

$t = 0$ & $t = 2$

↑ START ↑ AGAIN

AFTER 2S THE HEIGHT ABOVE GROUND WILL BE 48 FT. AGAIN.

Ⓒ (FIND THE Y-COOR OF THE V-TEX)

$h = -\frac{b}{2a} = \frac{-32}{2(-16)} = 1$

$h(1) = \underline{-16 + 32} + 48 = 16 + 48 = 64\text{ ft}$

MAX. HEIGHT IS **64 ft**

Ⓓ $h(1.5) = -16(1.5)^2 + 32(1.5) + 48 = 60\text{ft}$

Ⓔ WHEN IT LANDS, $h(t) = 0$.

$h(t) = 0$, SOLVE FOR t

$-16t^2 + 32t + 48 = 0$

$-16(t^2 - 2t - 3) = 0$

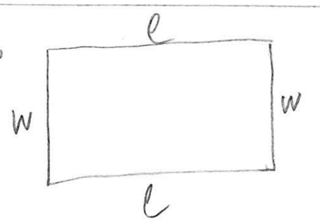
$-16(t-3)(t+1) = 0$

$\boxed{t = 3}$ OR $t = -1$

↑ NO NEG. TIME

IT WILL LAND 3S AFTER IT WAS LAUNCHED.

10.



$2w + 2l = 400$

$w + l = 200$

$\Rightarrow \boxed{l = 200 - w}$

Ⓐ $A = l \cdot w$

$A = (200 - w)w$

$\boxed{A(w) = 200w - w^2}$

Ⓑ "FIND THE "x" COOR. OF THE V-TEX"

$-\frac{b}{2a} = -\frac{200}{2(-1)} = 100$

LARGEST AREA FOR **W = 100m**

Ⓒ MAX AREA (Y COOR. OF V-TEX)

$A(100) = (200 - 100)100$

$= 100(100)$

$= \boxed{10000\ m^2}$

3. POLYNOMIAL FUNCTIONS

Ⓐ ALL REAL #'S

Ⓑ $X = a$ IS A ZERO OF A POL'Y IF $P(a) = 0$
(X-VALUE FOR WHICH THE OUTPUT IS 0)
(X-INTS)

Ⓒ THE GRAPH CAN EITHER TOUCH OR CROSS THE X-AXIS AT THE REAL ZEROS

Ⓓ THE MULTIPLICITY MUST BE ODD

Ⓔ HOW THE FN. BEHAVES WHEN $|X| \to \infty$

Ⓕ A POINT AT WHICH THE GRAPH SWITCHES FROM INCREASING TO DECREASING OR VICE VERSA.

Ⓖ $n-1$

Ⓗ THE GRAPH IS <u>BELOW</u> THE X-AXIS.

11. $f(x) = (x-4)(x+2)^2(x-2)$

Ⓐ $n = 4$

Ⓑ POSITIVE

Ⓒ  AS $X \to \pm\infty$, $f(x) \to +\infty$

Ⓓ YES IT IS

Ⓔ/Ⓕ/Ⓖ

ZEROS	MULTIP	C/T
$X = -2$	$m = 2$	T
$X = 2$	$m = 1$	C
$X = 4$	$m = 1$	C

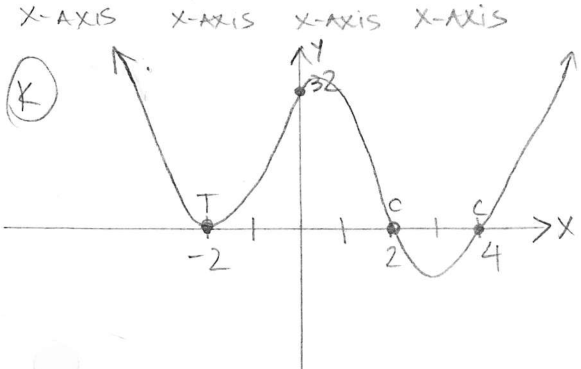
Ⓗ Y-INT $f(0) = (-4)(2)^2(-2) = 32$
$\boxed{(0,32)}$

Ⓘ $4 - 1 = 3$

Ⓙ

+	+	−	+
	−2	2	4
ABOVE X-AXIS	ABOVE X-AXIS	BELOW X-AXIS	ABOVE X-AXIS

Ⓚ



$f(x) = -4x^3 + 4x$

Ⓐ $n = 3$

Ⓑ NEG.

Ⓒ  AS $X \to +\infty$, $f(x) \to -\infty$
AS $X \to -\infty$, $f(x) \to +\infty$

Ⓓ $f(x) = -4x(x^2-1) = -4x(x-1)(x+1)$
$f(x) = -4x(x-1)(x+1)$ ← <u>NOW IT IS</u>

Ⓔ/Ⓕ/Ⓖ:

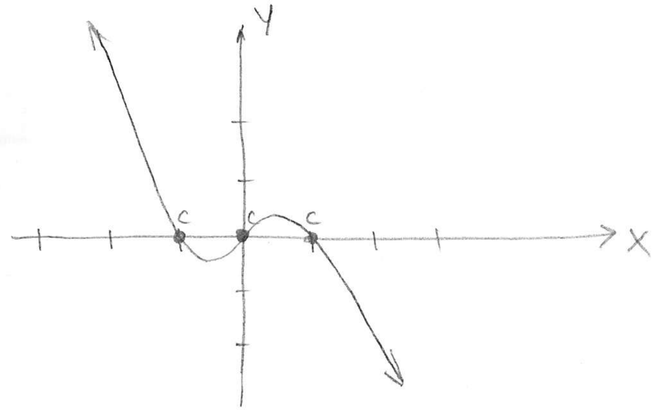
ZEROS	MULTIP	C/T
$X = -1$	$m = 1$	C
$X = 0$	$m = 1$	C
$X = 1$	$m = 1$	C

Ⓗ $(0,0)$ Ⓘ $3 - 1 = 2$

Ⓙ

+	−	+	−
	−1	0	1

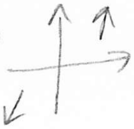
Ⓚ



11. C $f(x) = x(x-1)^2(x+3)(x+1)$

A $n = 5$

B POSITIVE

C 

AS $x \to +\infty$, $f(x) \to +\infty$

AS $x \to -\infty$, $f(x) \to -\infty$

D YES

E F G:

ZEROS	MULTIP	C/T
$x = -3$	$m = 1$	C
$x = -1$	$m = 1$	C
$x = 0$	$m = 1$	C
$x = 1$	$m = 2$	T

H $(0,0)$

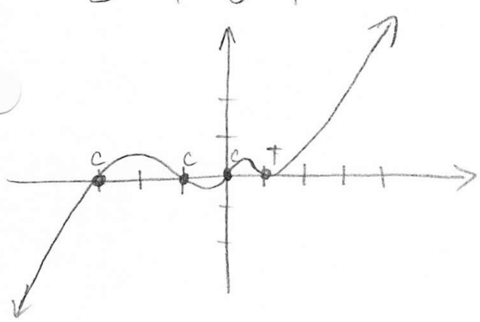
I $5 - 1 = 4$

J

$-$	$+$	$-$	$+$	$+$

$-3 \quad -1 \quad 0 \quad 1$

K



D $h(x) = (x^2 - 25)(x+5)$

A $n = 3$

B POSITIVE

C 

AS $x \to +\infty$, $h(x) \to +\infty$

AS $x \to -\infty$, $h(x) \to -\infty$

D NO!

$h(x) = (x-5)(x+5)(x+5) = (x-5)(x+5)^2$

$h(x) = (x+5)^2(x-5)$ ✓

E F G

ZEROS	MULTIP	C/T
$x = -5$	$m = 2$	T
$x = 5$	$m = 1$	C

H $h(0) = (-25)(5) = -125$ $\quad (0, -125)$

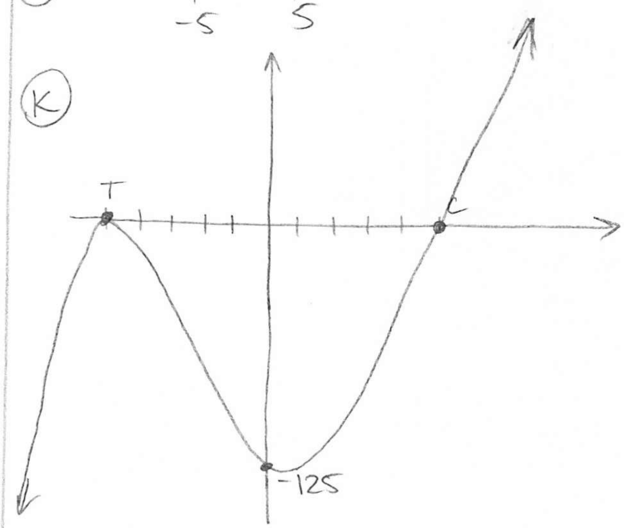
I $3 - 1 = 2$

J

$-$	$-$	$+$

$-5 \quad 5$

K



11. (E) $g(x) = -2x^4 + 4x^2$

(A) $n = 4$

(B) NEGATIVE

(C) AS $x \to \pm\infty$, $g(x) \to -\infty$

(D) NO

$g(x) = -2x^2(x^2 - 2)$

$g(x) = -2x^2(x - \sqrt{2})(x + \sqrt{2})$

(E)(F)(G)

ZEROS	MULT.	C/T
$x = -\sqrt{2}$	$m = 1$	C
$x = 0$	$m = 2$	T
$x = \sqrt{2}$	$m = 1$	C

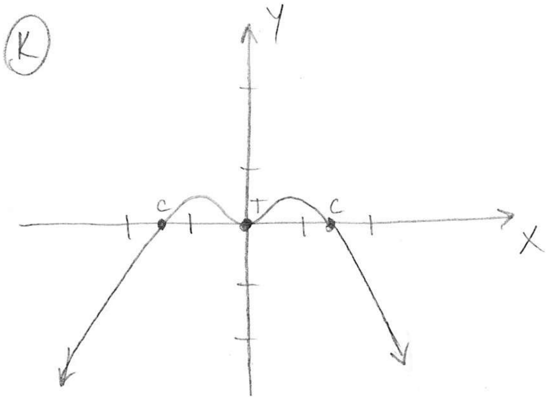
(H) $(0,0)$

(I) $4 - 1 = 3$

(J)

$-\sqrt{2}$		0		$\sqrt{2}$	
−	+		+		−

(K)



(F) $r(x) = \frac{1}{2}x^2(x^2 - 9)^2$

(A) $n = 6$

(B) POS.

(C) AS $x \to \pm\infty$, $r(x) \to +\infty$

(D) $r(x) = \frac{1}{2}x^2\left((x-3)(x+3)\right)^2 = \frac{1}{2}x^2(x-3)^2(x+3)^2$

(E)(F)(G)

ZEROS	MULT	C/T
$x = -3$	$m = 2$	T
$x = 0$	$m = 2$	T
$x = 3$	$m = 2$	T

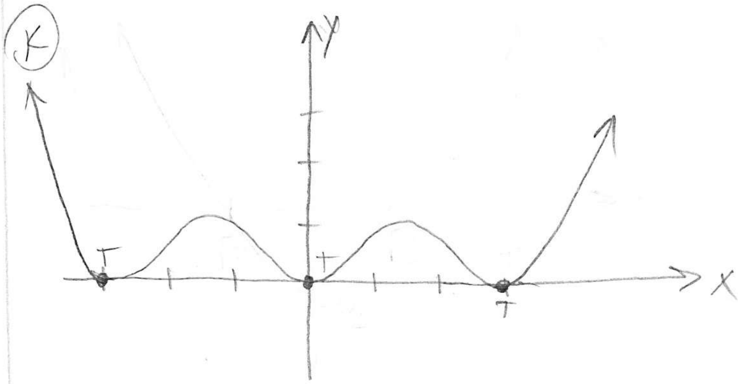
(H) $r(0) = 0$ $(0,0)$

(I) $6 - 1 = 5$

(J)

-3		0		3	
+	+		+		+

(K)



12. $P(x) = -3x^3(x+2)^2(x-7) = -3x^3(x^2 + 4x + 4)(x - 7)$

DEGREE: 6

$= (-3x^5 - 12x^4 - 12x^3)(x - 7)$

$= -3x^6 + 21x^5 - 12x^5 + 84x^4 - 12x^4 + 84x^3$

$= -3x^6 + 9x^5 + 72x^4 + 84x^3$

4. RATIONAL FUNCTIONS

A) $D: \{x \mid Q(x) \neq 0\}$

B) THE END BEHAV. OF THE GRAPH

C) HOW THE GRAPH BEHAVES AROUND THE VALUES FOR WHICH $Q(x)=0$ BUT $P(x) \neq 0$.

D) HORIZONTAL:
WHEN: THE DEG. OF THE NUM < DEG. OF DEN
OR THE DEG OF THE NUM = DEG. OF DEN.
SLANT
WHEN: THE DEG. OF NUM = DEG OF DEN. + 1
VERTICAL
WHEN $Q(x)=0$ BUT $P(x) \neq 0$.

SEE NOTES/BOOK

E) $y=3$ IS A H.A.

F) $x=3$ IS A V.A

G) SEE D

H) WHEN THE NUMERATOR & THE DENOM. HAVE COMMON FACTORS

I) NO. IT CAN HAVE EITHER HORIZ. OR SLANT & NO MORE THAN ONE

J) YES

K) IT CAN CROSS A HORIZ. OR A SLANT ASYM BUT NOT A VERTICAL ASY.

13. A) $f(x) = \dfrac{2x^2+7x+3}{x+1} = \dfrac{(2x+1)(x+3)}{x+1}$

V.A: $x=-1$

S.A:

$$\begin{array}{r} 2x+5-\frac{2}{x+1} \\ x+1 \,\big)\, 2x^2+7x+3 \\ -\ (\cancel{2x^2}+2x) \\ \hline 5x+3 \\ -\ (\cancel{5x}+5) \\ \hline -2 \end{array}$$

S.A: $y=2x+5$

NO H.A

NO HOLES

B) $g(x) = \dfrac{2x^2+1000000+x}{3x-x^2}$

$g(x) = \dfrac{2x^2+x+1000000}{-x^2+3x}$

$= \dfrac{2x^2+x+1000000}{-x(x-3)}$

V.A: $x=0$
$x=3$

H.A: $y=-2$ (DEG OF NUM = 2, DEG. OF DEN = 2 $\Downarrow$ H.A, $y = \dfrac{\text{LEAD. COEFF OF NUM}}{\text{LEAD COEFF OF DEN}}$)

NO SLANT

NO HOLES

C) $h(x) = \dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x-2)(x+2)}$

$\tilde{h}(x) = \dfrac{1}{x-2}$ (REDUCED FN)

V.A: $x=2$

H.A: $y=0$

HOLE: AT $x=-2$

$f(-2) = \dfrac{1}{-2-2} = -\dfrac{1}{4}$

$\boxed{(-2, -1/4)}$

D) $F(x) = \dfrac{x^2-4}{x+2} = \dfrac{(x+2)(x-2)}{x+2}$

$\tilde{F}(x) = x-2$ (REDUCED FN)

NO V.A

NO H.A

HOLE: AT $x=-2$

$\tilde{f}(-2) = -4$

$\boxed{(-2, -4)}$

S.A: $y=x-2$

$$\begin{array}{r} x-2 \\ x+2 \,\big)\, x^2-4 \\ -\ (\cancel{x^2}+2x) \\ \hline -2x-4 \\ -\ (-2x-4) \\ \hline 0 \end{array}$$

13. (E) $G(x) = \dfrac{10000x^2+3x-1}{x^3}$

V.A: $x=0$

H.A: $y=0$

NO SLANT

NO HOLES

(F) $R(x) = \dfrac{3x^4+3}{-7x+2x^4} = \dfrac{3(x^4+1)}{x(2x^3-7)}$

V.A: $x=0$ & $x=\sqrt[3]{7/2}$ H.A: $y=\dfrac{3}{2}$

$2x^3-7=0$

$2x^3=7$

$x^3=7/2$

$x=\sqrt[3]{7/2}$

NO SLANT

NO HOLES

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14. (A) $f(x) = \dfrac{2x^2+2x-12}{x^2-x-6} = \dfrac{2(x-2)(x+3)}{(x-3)(x+2)}$

(A) D: $\{x \mid x \neq -2 \text{ AND } x \neq 3\}$

(B) X-INT ($y=0$)

$2(x-2)(x+3)=0$

$x=2$ OR $x=-3$

$(2,0)$ & $(-3,0)$

(C) Y-INT: $y=\frac{-12}{-6}=2$

$(0,2)$

(D) VA: $x=-2$

$x=3$

HA: $y=2$

(E) TO CHECK IF THE GRAPH CROSSES THE H.A.:

$2 \stackrel{?}{=} \dfrac{2x^2+2x-12}{x^2-x-6}$

$2(x^2-x-6) = 2x^2+2x-12$

$\cancel{2x^2}-2x-12 = \cancel{2x^2}+2x-12$

$-4x=0$

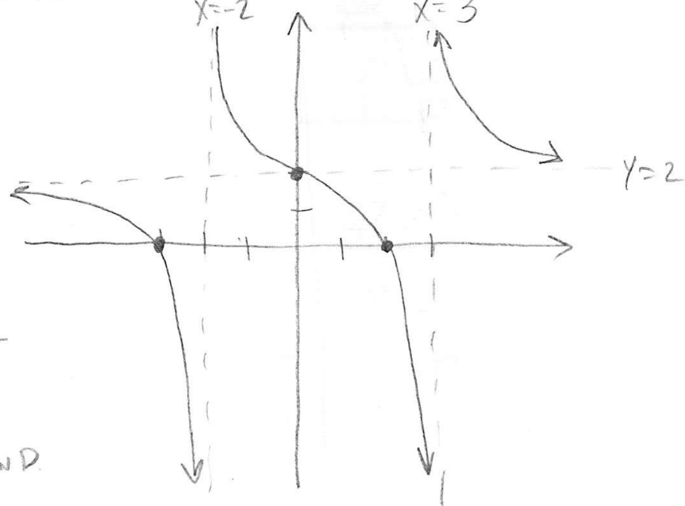
$x=0$

$\Rightarrow$ YES! AT $x=0$.

+	−	+	−	+

-3 -2 2 3

X-INT NOT IN D X-INT NOT IN D



14. (B) $f(x) = \frac{x^3}{x^2-4} = \frac{x^3}{(x-2)(x+2)}$

D: $\{x \mid x \neq \pm 2\}$

ZEROS: NUM $\stackrel{SET}{=} 0$

$x^3 = 0$ $(0,0)$

$x = 0$

Y-INT: $(0,0)$

V.A: $x = -2$

$x = 2$

S.A: $y = x$

$$x^2-4 \overline{)\,x^3} \quad x + \frac{4x}{x^2-4}$$

$$\cancel{x^3} - 4x$$

$$- \quad +$$

$$4x$$

NO HA

NO HOLES

CHECK IF THE GRAPH CROSSES THE SA:

$x \stackrel{?}{=} \frac{x^3}{x^2-4}$

$x^3 - 4x = x^3$

$\cancel{x^3} - \cancel{x^3} - 4x = 0$

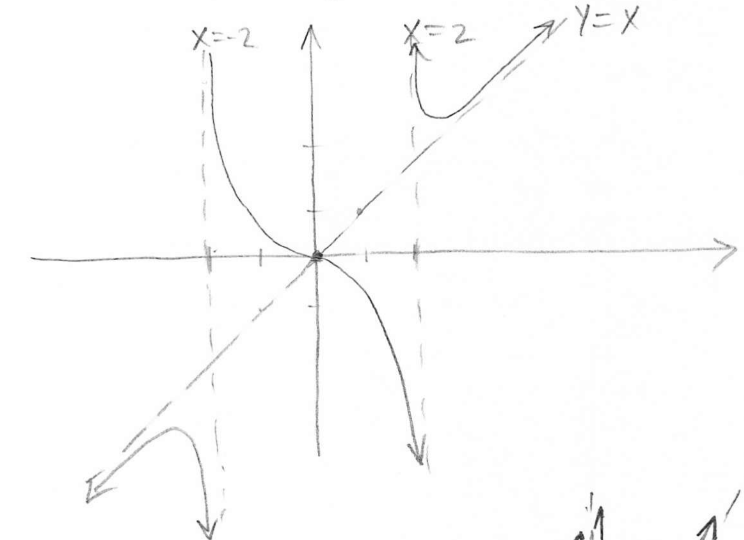
$x = 0$

YES AT $\underline{\underline{x = 0}}$

BELOW X-AXIS

$-$ | -2 | $+$ | 0 | $-$ | 2 | $+$

ABOVE X-AXIS



(C) $f(x) = \frac{3x^3}{(x-1)^2} = \frac{3x^3}{x^2-2x+1}$

(A) $\{x \mid x \neq 1\}$

(B) X-INT: $3x^3 = 0 \Rightarrow x = 0$

X,Y-INT: $(0,0)$

(D) V.A: $x = 1$

S.A:

$$x^2-2x+1 \overline{)\,3x^3} \quad 3x + 6 + \frac{9x-6}{x^2-2x+1}$$

$$\cancel{3x^3} - 6x^2 + 3x$$

$$- \quad + \quad -$$

$$6x^2 - 3x$$

$$\cancel{6x^2} - 12x + 6$$

$$- \quad + \quad -$$

$$9x - 6$$

SA: $y = 3x + 6$

(E) TO CHECK IF CROSSES THE SA

$3x + 6 \stackrel{?}{=} \frac{3x^3}{x^2-2x+1}$

$(3x+6)(x^2-2x+1) = 3x^3$

$\cancel{3x^3} - \cancel{6x^2} + 3x + \cancel{6x^2} - 12x + 6 = \cancel{3x^3}$

$-9x + 6 = 0$

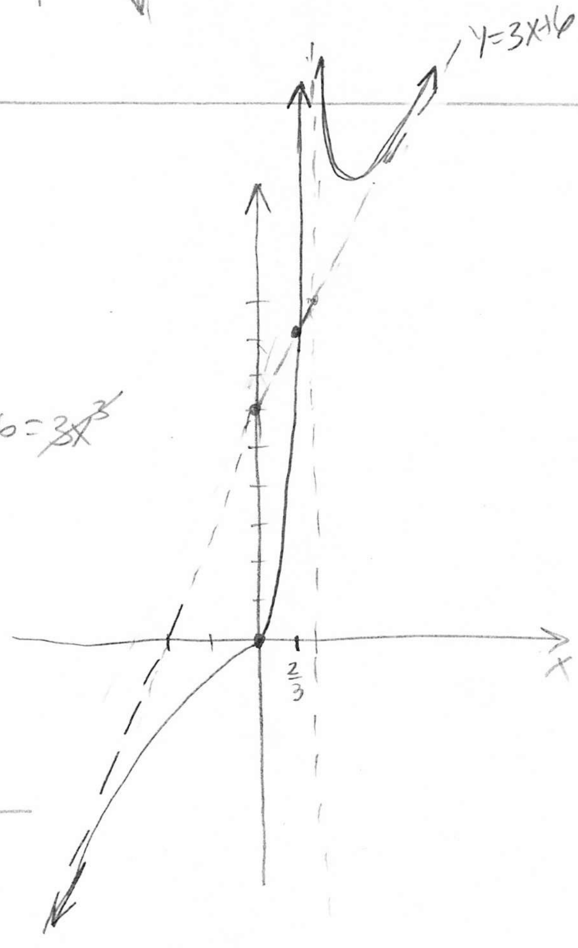
$9x = 6$

$x = 6/9 = 2/3$

YES! AT $x = 2/3$

(F)

$-$ | 0 | $+$ | 1 | $+$



14. D $f(x) = \frac{(x-1)^2}{x^2-1} = \frac{(x-1)^2}{(x-1)(x+1)}$

DOMAIN: $\{x \mid x \neq \pm 1\}$

SIMPLIFIED FN: $\tilde{f}(x) = \frac{x-1}{x+1}$

* THE GRAPHS OF f & $\tilde{f}$ WILL BE THE SAME EVERYWHERE <u>EXCEPT</u> AT <u>$x=1$</u>

X-INT: (SET $y=0$)

$\Rightarrow$ NUM $= 0$

$(x-1)^2 = 0$

$x = 1$

<u>BUT</u> $x=1$ IS NOT IN THE DOMAIN

$\Rightarrow$ NO X-INTERCEPTS

INTERVALS IN WHICH IT IS ABOVE/BELOW THE X-AXIS:

+	-1	$-$	1	+
ABV.		BELOW		ABV.

Y-INT (SET $x=0$)

$f(0) = \frac{-1}{1} = -1$

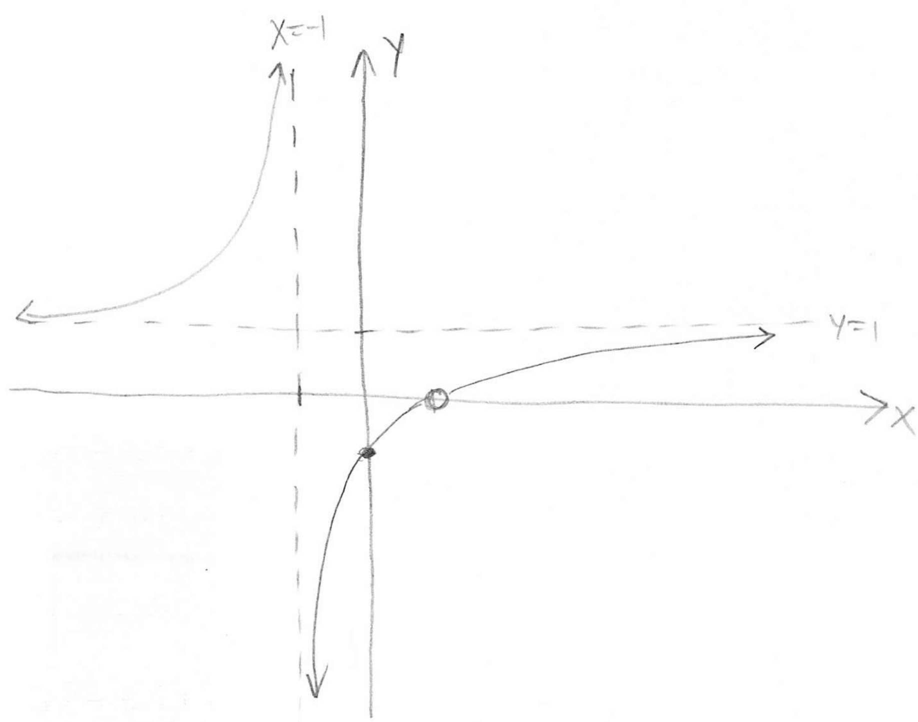
$\boxed{(0,-1)}$

$\boxed{\begin{array}{l} V.A: x=-1 \\ H.A: y=1 \end{array}}$

HOLE: AT $x=1$

$\tilde{f}(1) = \frac{1-1}{1+1} = 0$

$\boxed{(1,0)}$



CHECK IF THE GRAPH CROSSES THE HORIZ ASYMPTOTE:

HA: $y=1$

FN: (WE CAN USE $\tilde{f}(x)$)

$1 \stackrel{?}{=} \tilde{f}(x)$

$1 = \frac{x-1}{x+1}$

$x+1 = x-1$

$x - x = -1-1$

$0 \neq -2$ <u>NO IT DOES NOT</u>

14. (E) $f(x) = \dfrac{x^4}{x^2-9} = \dfrac{x^4}{(x-3)(x+3)}$

D: $\{x \mid x \neq \pm 3\}$

ZEROS: $x^4 = 0$ $\quad (0,0)$

$x = 0$

X/Y-INT

V.A: $x = 3$

$x = -3$

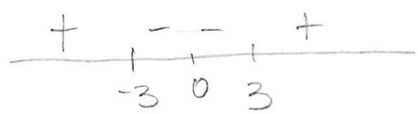
NO H.A

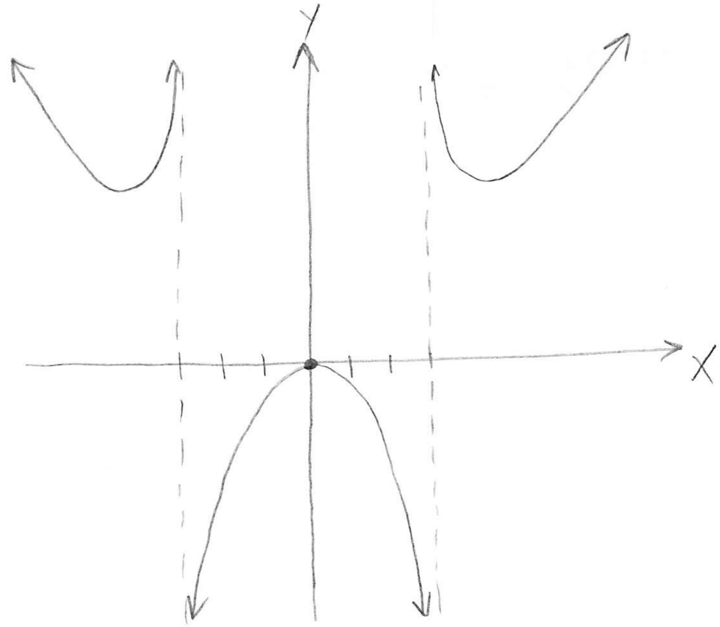
NO S.A

NO HOLES

AS $x \to +\infty$, $f(x) \to +\infty$

AS $x \to -\infty$, $f(x) \to +\infty$





(F) $f(x) = \dfrac{x^2-9}{x^2-2x-3} = \dfrac{(x-3)(x+3)}{(x-3)(x+1)}$

D: $\{x \mid x \neq -1 \text{ AND } x \neq 3\}$

SIMPLIFIED FN: $\tilde{f}(x) = \dfrac{x+3}{x+1}$

X-INT: $x+3=0$

$x=-3$ $\quad \boxed{(-3,0)}$

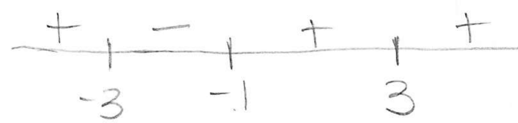
Y-INT: $(x=0)$ $\quad y = \dfrac{-9}{-3} = 3$ $\quad \boxed{(0,3)}$

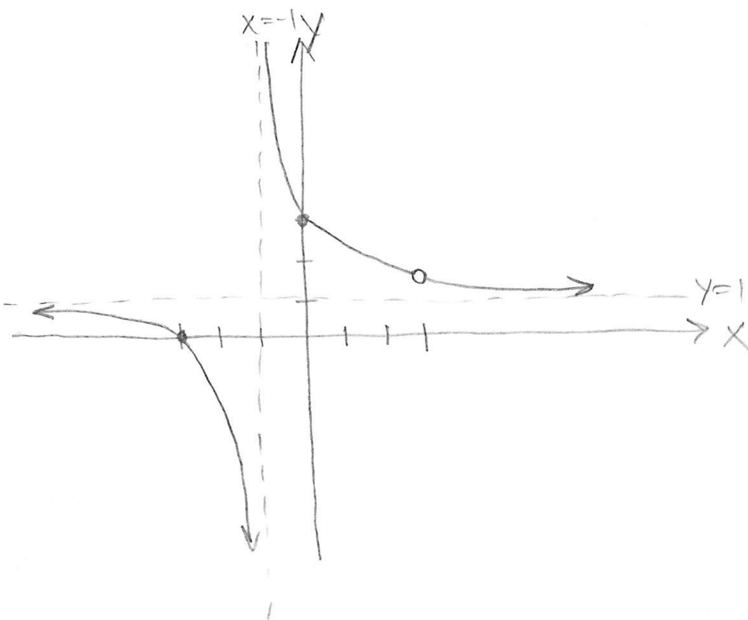
VA: $x=-1$

H.A: $y=1$

HOLE: AT $x=3$

$\tilde{f}(3) = \dfrac{6}{4} = \dfrac{3}{2}$ $\quad (3, 3/2)$





CHECK IF THE GRAPH CROSSES THE HA:

$1 \stackrel{?}{=} \dfrac{x+3}{x+1}$

$x+1 \stackrel{?}{=} x+3$

$x - x \stackrel{?}{=} 3-1$

$0 \neq 2$ (FALSE STATEM)

=> NO IT DOES NOT

14. (G) $f(x) = \dfrac{(x+2)(x^2-2x+1)}{x^2+5x+6} = \dfrac{(x+2)(x-1)^2}{(x+3)(x+2)}$

D: $\{x \mid x \neq -3 \text{ AND } x \neq -2\}$

SIMPLIFIED FN: $\tilde{f}(x) = \dfrac{(x-1)^2}{x+3} = \dfrac{x^2-2x+1}{x+3}$

HOLE: $(-2, 9)$

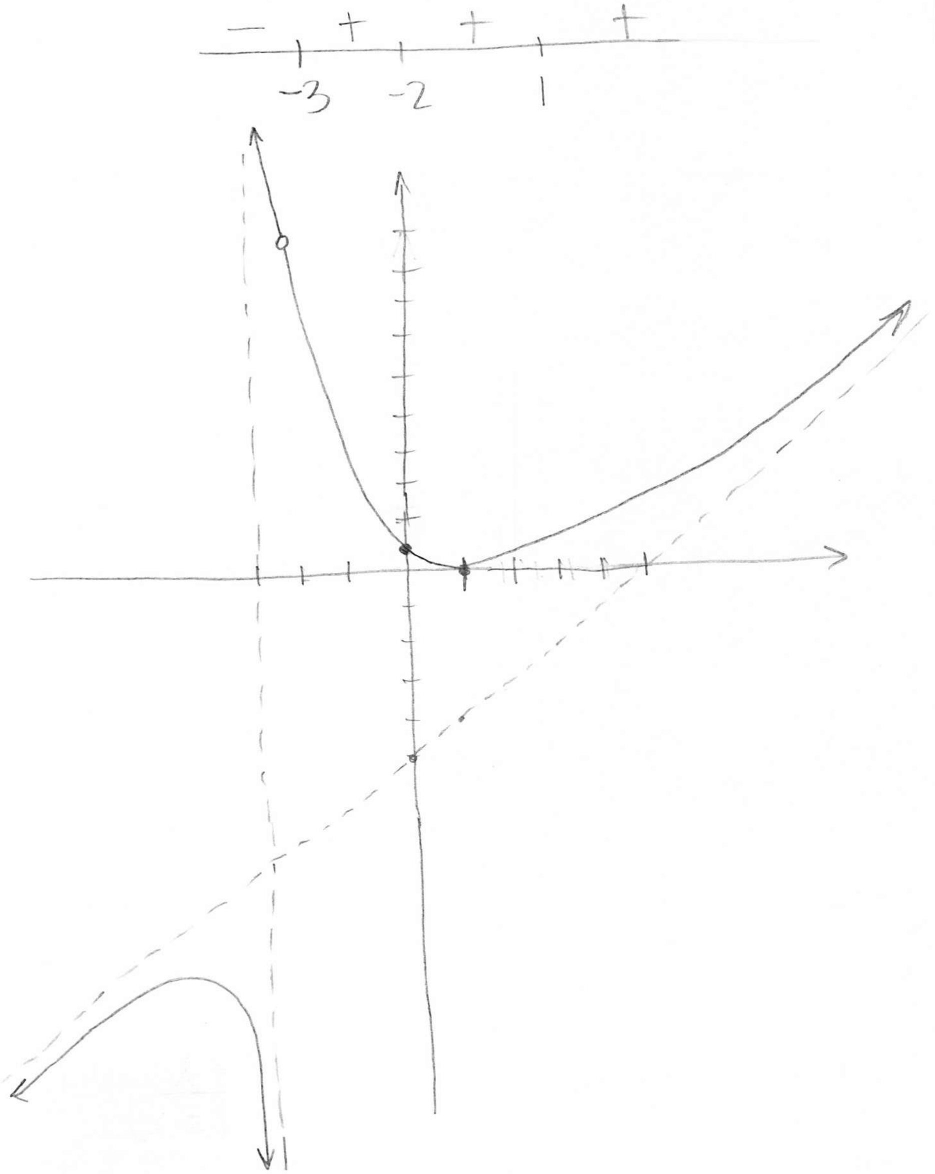
$\tilde{f}(-2) = \dfrac{(-2-1)^2}{-2+3} = \dfrac{9}{1}$

X-INT: $(x-1)^2 = 0$ $\quad \boxed{(1,0)}$

$x = 1$

Y-INT $(x=0)$: $y = \dfrac{2(1)}{6} = \dfrac{1}{3}$

$\boxed{\left(0, \tfrac{1}{3}\right)}$



V.A: $x = -3$

SA:

$$\begin{array}{r} x - 5 + \frac{16}{x+3} \\ x+3 \enclose{longdiv}{x^2 - 2x + 1} \\ -(x^2 + 3x) \\ \hline -5x + 1 \\ -(-5x - 15) \\ \hline 16 \end{array}$$

$\boxed{y = x - 5}$ SA

CHECK IF GRAPH CROSSES SA:

$x - 5 \stackrel{?}{=} \dfrac{x^2-2x+1}{x+3}$

$(x-5)(x+3) \stackrel{?}{=} x^2-2x+1$

$\cancel{x^2} - \cancel{2x} - 15 = \cancel{x^2} - \cancel{2x} + 1$

$-15 \neq +1$

NO IT DOES NOT!

---

15. (A) $f(x) = \dfrac{7}{x-3}$

(B) $g(x) = \dfrac{1}{x}$

(C) $h(x) = \dfrac{x^2+3}{(x-2)(x-5)} = \dfrac{x^2+3}{x^2-7x+10}$

(D) $R(x) = \dfrac{117x}{x^2+1}$

(☆) ANSW NOT UNIQUE

16. (A)

$x^2 < x$

$x^2 - x < 0$

$x(x-1) < 0$

ZEROS: $x = 0$

$x = 1$

$+ \quad (-) \quad +$

$0 \qquad 1$

ANSW: $(0,1)$

(B) $\underbrace{x^2+1}_{} \le 0$

NO ZEROS

$+$

LHS ALWAYS (+)VE

NO S/N

(C) $x^2+2x \ge -1$

$x^2+2x+1 \ge 0$

$(x+1)^2 \ge 0$

$+ \quad +$

-1

ANSW: $(-\infty, \infty)$

(D) $|x+5| - 5 < 7$

$|x+5| < 7+5$

$|x+5| < 12$

$-12 < x+5 < 12$

$-5 \quad -5 \quad -5$

$-17 < x < 7$

S/N: $(-17, 7)$

(E) $2|7-x| + 1 > 4$

$2|7-x| > 4-1$

$2|7-x| > 3$

$|7-x| > \frac{3}{2}$

$7 - x < -\frac{3}{2}$ OR $7 - x > \frac{3}{2}$

$-x < -\frac{3}{2} - 7 \qquad -x > \frac{3}{2} - 7$

$-x < -\frac{17}{2} \qquad -x > -\frac{11}{2}$

$x > \frac{17}{2} \qquad x < \frac{11}{2}$

S/N:

$\frac{11}{2} \qquad \frac{17}{2}$

$(-\infty, \frac{11}{2}) \cup (\frac{17}{2}, \infty)$

(F) $x^2 \le 3 - 2x$

$x^2 + 2x - 3 \le 0$

$(x+3)(x-1) \le 0$

ZEROS: $x = -3$

$x = 1$

$+ \quad (-) \quad +$

$-3 \qquad 1$

S/N: $[-3, 1]$

(H) $5x + 4 < 3x^2$

$-3x^2 + 5x + 4 < 0$

$3x^2 - 5x - 4 > 0$

ZEROS:

$x = \frac{5 \pm \sqrt{25+48}}{6}$

$= \frac{5 \pm \sqrt{73}}{6}$

$(+) \quad - \quad (+)$

$\frac{5-\sqrt{73}}{6} \qquad \frac{5+\sqrt{73}}{6}$

S/N: $\left(-\infty, \frac{5-\sqrt{73}}{6}\right) \cup \left(\frac{5+\sqrt{73}}{6}, \infty\right)$

(G) $5x + 4 \ge 3x^2$

$-3x^2 + 5x + 4 \ge 0$

$3x^2 - 5x - 4 \le 0$

SAME ZEROS (USE SAME # LINE)

INTERESTED IN THE $-$'s:

$\left[\frac{5-\sqrt{73}}{6}, \frac{5+\sqrt{73}}{6}\right]$

(I) $\left|\frac{11}{4}x - 3\right| \ge \frac{15}{4}$

$\frac{11x}{4} - 3 \le -\frac{15}{4}$ OR $\frac{11x}{4} - 3 \ge \frac{15}{4}$

$\frac{11x}{4} \le -\frac{15}{4} + 3 \qquad \frac{11x}{4} \ge \frac{15}{4} + 3$

$\frac{11x}{4} \le -\frac{3}{4} \qquad \frac{11x}{4} \ge \frac{27}{4}$

$x \le -\frac{3}{\cancel{4}} \cdot \frac{\cancel{4}}{11} \qquad x \ge \frac{27}{\cancel{4}} \cdot \frac{\cancel{4}}{11}$

$x \le -\frac{3}{11} \qquad x \ge \frac{27}{11}$

$-\frac{3}{11} \qquad \frac{27}{11}$

S/N: $(-\infty, -\frac{3}{11}] \cup [\frac{27}{11}, \infty)$

(J) $\dfrac{(x-2)(x-1)}{x+3} \geq 0$

ZEROS: $x=1$
$x=2$

UNDEF FOR: $x=-3$

$-$ $+$ $-$ $+$

-3 1 2

SIN: $(-3,1] \cup [2,\infty)$

(K) $\dfrac{x+1}{x(x+3)} \leq 0$

ZEROS: $x=-1$

UND: $x=0$
$x=-3$

$-$ $+$ $-$ $+$

-3 -1 0

SIN: $(-\infty,-3) \cup [-1,0)$

(L) $\dfrac{6}{x+3} \geq 0$

NO ZEROS

UND: $x=-3$

$-$ $+$

-3

SIN: $[-3,\infty)$

(M) $\dfrac{6}{x+3} \geq 1$

$\dfrac{6}{x+3} - 1 \geq 0$

$\dfrac{6}{x+3} - \dfrac{x+3}{x+3} \geq 0$

$\dfrac{6-(x+3)}{x+3} \geq 0$

$\dfrac{3-x}{x+3} \geq 0$

UND: $x=-3$

ZEROS: $x=3$

$+$ $+$ $-$

-3 3

SIN: $(-\infty,-3) \cup (-3,3]$

(N) $\dfrac{6}{x+3} < 1$

SIMMILAR

⋮

$\dfrac{3-x}{x+3} < 0$

(SAME #'LINE)

INTERESTED IN $-$'s

$+$ $+$ $-$

-3 3

SIN: $(3,\infty)$

(O) $\dfrac{x^2-8x+12}{x^2-16} < 0$

$\dfrac{(x-6)(x-2)}{(x-4)(x+4)} < 0$

ZEROS: $x=6,2$

UND: $x=\pm 4$

$+$ $-$ $+$ $-$ $+$

-4 2 4 6

SIN: $(-4,2) \cup (4,6)$

(P) $\dfrac{6}{x^2+3} \geq 0$

NO ZEROS

DEFINED $\forall x$

$+$

SIN: $(-\infty,\infty)$

(Q) $\dfrac{3x-1}{x^2+1} \geq 1$

$\dfrac{3x-1}{x^2+1} - 1 \geq 0$

$\dfrac{3x-1}{x^2+1} - \dfrac{x^2+1}{x^2+1} \geq 0$

$\dfrac{3x-1-(x^2+1)}{x^2+1} \geq 0$

$\dfrac{-x^2+3x-2}{x^2+1} \geq 0$

$-\dfrac{(x-2)(x-1)}{x^2+1} \geq 0$

ZEROS: $x=2,1$

DEFINED $\forall x$

$-$ $+$ $-$

1 2

SIN: $[1,2]$

(R) $\dfrac{2x+17}{x+1} \geq x+5$

$\dfrac{2x+17}{x+1} - (x+5) \geq 0$

$\dfrac{2x+17}{x+1} - \dfrac{(x+5)(x+1)}{x+1} \geq 0$

$\dfrac{2x+17-(x^2+6x+5)}{x+1} \geq 0$

$\dfrac{-x^2-4x+12}{x+1} \geq 0$

$\dfrac{-(x+6)(x-2)}{x+1} \geq 0$

ZEROS: $x=-6,2$

UND: $x=-1$

$+$ $-$ $+$ $-$

-6 -1 2

SIN: $(-\infty,-6] \cup (-1,2]$