STAT 345 Spring 2018
Homework 1 - Set Theory and Counting Name: _____

Please adhere to the homework rules as given in the Syllabus.

**1. Sample Spaces.** For each of the following experiments, (1) give the Sample Space using set notation, (2) determine whether each sample space is *discrete* or *continuous* and (3) give an example of a non-trivial event..

a) Experiment: Draw a card at random from a standard deck.
Outcome: The suit of the chosen card.

b) Experiment: Alf gets a Tinder for 1 week.
Outcome: The number of matches Alf gets.

c) Experiment: Michael Phelps swims the 200m Butterfly.
Outcome: The time it takes him to finish.

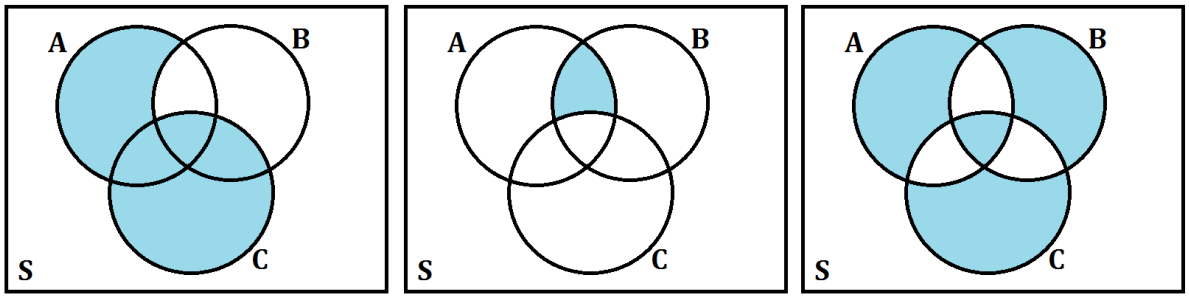
**2.** **Venn Diagrams.** For each part below, create a 3-set Venn Diagram (with a border representing $S$) and shade in the region corresponding to the given event.

a) $(A \cap B)^c$

b) $(A \cup B) \cap C$

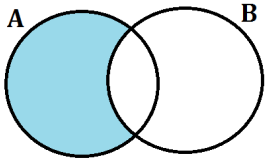
c) $(A \cup B \cup C) \cap (A \cap B \cap C)^c$

**3.** **More Venn Diagrams!** For each of the three venn diagrams below, give an expression for the shaded region using only unions, intersections and complements.

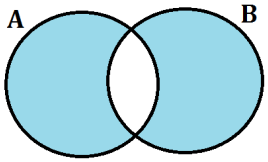


**4. Set Operations.**

**a)** The difference (subtraction) operator $A - B$ reads "$A$ minus $B$" is illustrated in the venn diagram below. Give an equivalent expression for $A - B$ using only unions, intersections and complements.




**b)** The symmetric difference operator $A \triangle B$ is illustrated in the venn diagram below. Note that this is similar to the logical idea of an *exclusive or*. Give an equivalent expression for $A \triangle B$ using only unions, intersections and complements.




**5.** A basketball team will have 5 players on the court at all times. A certain team has 12 players on their roster.

a) How many different combinations of players can the team have on the court at a given time?

b) There are 5 distinct positions in basketball. How many different ways can the team choose a lineup of 5 players. (*The key difference here is that the lineup (Alf, Bob, Carl, Dudley, Emma) is NOT the same as the lineup (Emma, Dudley, Carl, Bob, Alf) because they are playing different positions).*

**6.** Professor Halfbrain has 6 books on mathematics and 5 books on chemistry. He wants to place a total of 4 books on a shelf, with the requirement that books of the same subject should be next to each other and in alphabetic order. For example, these are both valid orderings.

- Math by A  Math by C  Math by Z  Chem by B
- Chem by B  Chem by Z  Math by C  Math by Z

How many different ways can he arrange the books?
*Hint: Consider the partition* $B_i = \{PH \text{ puts } i \text{ math books on the shelf}\}$ *for* $i = 0, 1, \cdots 4$.

**7.** A bit is either a 0 or a 1. A *byte* is a sequence of 8 bits and a *nibble* is a sequence of 4 bits.

a) How many different bytes are possible?

b) What is the probability that a randomly generating byte has exactly 3 ones?

c) How many different nibbles are possible? Write out the sample space.

d) What is the probability that a randomly generated nibble has two adjacent 1's?

**8. Challenge Problem - March Madness.** March Madness is a single elimination tournament with 64 teams. Determine the number of distinct brackets (show work). *Hint: Figure out how many ways there are to choose winners at each round, and use the Fundamental Theorem of Counting to obtain your final answer.* Now assume that you have a computer program which generates distinct brackets *at random* a million times every second for 2 weeks. What is the probability that your computer program generates a perfect bracket?