STAT 345 Fall 2018
Homework 3 - Discrete Random Variables Name: ______________________

Please adhere to the homework rules as given in the Syllabus.

1. **Frequency Distribution** The total number of goals scored in a World Cup soccer match approximately follows the following distribution.

Goals Scored	0	1	2	3	4	5	6	7
Probability	0.1	0.2	0.25	0.2	0.15	0.06	0.03	0.01

a) Let X be the number of goals scored in a randomly selected World Cup soccer match. Write out the PMF for X and explain why it is a valid PMF.

b) Compute the mean and variance of X.

c) Find and sketch the CDF of X. Explain why it is a valid CDF.

2.

a) Recall that for a random variable X and constants a and b, $E(aX + b) = aE(X) + b$ (we proved this in class). Prove that

$$Var(aX + b) = a^2 Var(X)$$

b) Mike agrees to donate 50 dollars to charity, plus 25 dollars for every goal scored in a particular World Cup soccer game. Thus Mike's donation is a random variable $Y = 25X + 50$ where X is defined in problem 1. Find the expected value and the variance of Y using the results in part a.

3. Probability Mass Function. Let X be a discrete RV with the following PMF.

$$f(x) = \begin{cases} \frac{c}{2^x} & x = 1, 2, 3, 4 \\ 0, & otherwise \end{cases}$$

a) Find the value of c that makes $f(x)$ a valid PMF.

b) Find $P(X > 2)$.

c) Find $E(X)$ and $Var(X)$.

d) Find $E(2^X)$.

4. Linearity of Expectation. The SAT is a multiple choice exam, with 5 possible answers for each question. To discourage test takers from guessing, they use the following point system. If you get a question right you get 1 point. If you get a question wrong you lose 0.25 points. (If you leave a question blank, you get/lose nothing, but that doesn't matter for this problem).

a) Assuming you are completely guessing, what is the probability of getting the question correct?

b) We can model this using a Bernoulli random variable, where $X = 1$ if you get the question right and $X = 0$ if you get the question wrong. The number of points earned on a single question is a random variable and can be expressed as

$$Y = X - 0.25(1 - X) = 1.25X - 0.25$$

Use linearity of expectation to show that $E(Y) = 0$ if you are guessing at random.

c) Assume now that there are k possible answers for each question. In terms of k, how many points should we take off for a wrong answer if we want to ensure $E(Y) = 0$ when a test-taker is guessing? *Hint: Let $Y = X - L(1 - X)$ where L is the number of points we remove. Find the expected value and solve for L.*

5. **Expected Values.** Four buses are transporting 150 UNM students to Las Cruces for a football game. The buses carry, respectively, 20, 30, 40 and 60 students.

- A student is selected at random (call him Alf). Let X be the number of students on Alf's bus.
- A bus driver is selected at random (call her Betty). Let Y be the number of students on Betty's bus.

Find $E(X)$ and $E(Y)$. Which one is larger? Why?

6. **Get Rich Quick.** Professor Halfbrain has come up with a betting strategy for roulette which he guarantees will make you rich. Bet 100 dollars on red (which has probability $\frac{18}{38}$). If you win, take the 100 dollar profit and quit. If you lose, bet 100 dollars on red two more times and then quit. Let X denote your earnings.

a) Find the probability mass function of X. *Hint: There are only* 3 *possible values that* X *can take. Find the two easier probabilities, and find the third by using the fact that the probabilities must sum to* 1.

b) What is $P(X > 0)$. What do you think of the Professors scheme?

c) Find $E(X)$. What do you think of the Professors shceme now?

7. Challenge Problem. *The Labrynth.* You are trapped inside a chamber at the center of a Labrynth. You have 24 hours to escape before you run out of oxygen.

- This chamber has n tunnels leading out of it, the i^{th} of which takes i hours to travel (the first tunnel takes 1 hour, the second tunnel takes 2 hours, etc...).
- The longest tunnel (tunnel n) will lead you to safety, but the rest of the tunnels just bring you back to the chamber.
- Every time you return to the tunnel, the room shifts, so you can only choose tunnels at random.

Let X be the amount of time that it takes you to escape.

Hint: For part a), use the *Law of Total Expectation*: For a partition $B_1, B_2, \cdots B_k$,

$$E(X) = \sum_{i=1}^{k} E(X|B_i)P(B_i)$$

a) How many hours will it take you to escape on average? For what values of n will you escape, on average, before your oxygen runs out?

b) Markov's Inequality states that, for a non-negative random variable X and a postive constant t,

$$P(X \geq t) \leq \frac{E(X)}{t}$$

Use Markov's Inequality to obtain a lower bound on the probability that you escape before you run out of oxygen. (i.e. find a lower bound for $P(X \leq 24)$.) Evaluate and interpret this for $n = 7$ tunnels.