

Please adhere to the homework rules as given in the Syllabus.

1. Coin Flipping. Timothy and Jimothy are playing a betting game. Timothy will flip a fair coin until it lands Heads. If X is the total number of flips, then Jimothy has to pay Timothy $g(X)$ dollars, where $g(X)$ is defined as

$$g(X) = \begin{cases} -5, & X = 1 \\ 0, & X = 2, 3, 4 \\ M, & X \geq 5 \end{cases}$$

Using a computer, plot $E(g(X))$ as a function of M . For what value of M is the game "fair" in the sense that $E(g(X)) = 0$?

2. Build a Test Problem. For this problem, try really hard to **be creative**. Try to use an example that is from your life, work or make something up. Your examples can be humorous or not humorous. A bonus point will be given for unique questions.

Give an example of a Binomial random variable U and a Geometric random variable V . If possible, relate both of these RVs to the same Bernoulli trial. Ask between 2 and 4 test-type questions and give the answer (you don't have to show work). If somebody gives a good enough answer here, I might use it on the midterm. *Hint: You can look at question 6 on the Spring 2018 Midterm for an example of a test-type problem. Yours should be **significantly** different.*

3. Professor Halfbrain has just created a fair five sided die, with sides numbered 1 through 5. This die is rolled once, and we let X be the number facing up.

a) Determine the distribution, expected value and the variance of X .

Professor Halfbrain constructs two more of these fair five sided die, but he chooses to number these 2 through 6 and 3 through 7 respectively.

b) If the Professor throws all 3 dice, what is the probability that the sum of their numbers is equal to 7? *Hint: Approach this as a counting problem.*

c) All three die are placed inside of a bag. The Professor reaches into the bag, grabs a die at random and tosses it. Let Y be the number facing up. Find $P(Y > 5)$. *Hint: Use Law of Total Probability.*

4. JeBron Lames is a basketball player who makes his free throws 70% of the time. Whether or not he makes a free throw is independent of his previous shots. Suppose JeBron shoots 10 freethrows, and let X be the number of free throws that he makes.

a) Identify the distribution of X , and write out it's PMF.

b) Determine the expected value and the variance of X .

c) Find $P(X = 5)$ and $P(X > 2)$.

5. Legendary STAT 345 Instructor Zach Stuart plans to swipe right until he gets a match. The probability of a match is (sadly) quite low at 2%. Let X be the number of times Zach will swipe right.

a) Identify the distribution of X , and write out its PMF **and** its CDF.

b) Determine the expected value and the variance of X .

c) What is the probability that Zach get's a match before he runs out of likes (100 likes per day).

6. Walter White and Jesse Pinkman run an operation where they cook blue(berry) muffins. Let X be the number of blueberries found in one of their muffins and assume that X follows a Poisson distribution with mean λ .

- a) Determine the expected value and the variance of X .

- b) Assume $\lambda = 9$. One of Walter and Jesse's customers gets very angry when he gets a muffin with 0 blueberries. What is the probability that this happens?

- c) Blueberries are in short supply! Walter and Jesse want to reduce the average number of blueberries per muffin (λ) to save costs. What is the smallest value of λ possible, which would still ensure the probability in part *b*) is no more than 0.01?

- d) Assume now that the customer gets angry if he gets a muffin with less than or equal to 1 blueberry. For the same requirement in *c*), what is the smallest possible value of λ now? (*Note: You will not be able to solve for λ analytically. Use any method you like (including guess and check) to give a correct answer to 1 decimal place.*)

- e) Assume $\lambda = 9$. Let Y be the number of blueberries found in a dozen (12) of Walter and Jesse's muffins. What is the distribution, mean and variance of Y ?

7. Challenge Problem: Refer to the Bayes Theorem example we did in class, where Alf is being tested for a rare disease. Recall that, even if Alf tests positive the probability he has the disease is fairly small. Therefore Alf may need to take n tests instead of just one. If we let X be the number of times Alf tests positive, we are dealing with a Binomial distribution where the probability of success is conditional on whether or not Alf has the disease. Let $D = \{\text{Alf has the disease}\}$ and assume $P(D) = 0.00001$.

$$X|D \sim \text{Binom}(n, 0.99)$$

$$X|D^c \sim \text{Binom}(n, 0.05)$$

Now, assume that Alf tests positive for k tests ($k = 0, 1, \dots, n$). We can use Bayes Theorem as follows,

$$P(D|X = k) = \frac{P(X = k|D)P(D)}{P(X = k)}$$

Where $P(X = k) = P(X = k|D)P(D) + P(X = k|D^c)P(D^c)$ by Law of Total Probability.

a) Compute $P(X = k|D)$ and $P(X = k|D^c)$ using the appropriate Binomial PMF.

b) Use Bayes Theorem to give an explicit expression for $P(D|X = k)$ in terms of n and k .

c) If Alf takes $n = 4$ tests and tests positive $k = 3$ times, what is the probability he has the disease?

d) How many consecutive tests does Alf need to test positive for, before the probability he has the disease is at least 0.99?