STAT 345 Fall 2018
Homework 6 - Continuous Probability Distributions

Name: ________________________

Please adhere to the homework rules as given in the Syllabus.

1. **Standard Normal.** Let Z be a Standard Normal RV.

a) Find $P(Z < 1.55)$ and $P(Z > 1.55)$

b) Find $P(Z < -0.64)$ and $P(Z > 0.64)$

c) Find $P(1.55 < Z < 2.45)$.

d) Find $P(|Z| < k)$ for $k = 1, 2, 3$.

2. Let X be the height of a randomly selected UNM male student. Assume that X is normally distributed with mean $\mu = 70$ and standard deviation $\sigma = 3$.

a) Find the probability that a randomly selected UNM male student is taller than legendary rapper Lonzo Ball (6 ft 6 in).

b) Find the probability that a randomly selected UNM male student is between 5 feet and 6 feet tall.

c) If exactly 25% of UNM males are taller than Timothy, how tall is Timothy?

3. **The Log-normal Distribution.** In atmospheric science, the Log-normal distribution is often used to characterize particle size distributions. In one study, the distribution of silicone nanoparticle size (in nm) was found to be approximately Log-normal with Log-mean $\theta = 3.91$ and Log-sd $\omega = 0.47$. Suppose the desired range of particle sizes is $(40nm, 110nm)$. What percentage of silicone nanoparticles do you expect to fall within this range?

4. Normal Approximations.

a) Suppose that the number of cars that drive past Lomas on the I-25 between 5pm and 6pm on a Wednesday can be modeled as a Poisson random variable with mean $\lambda = 2100$. Use the Normal approximation to the Poisson distribution to determine the probability that less than 2000 cars drive past Lomas on the I-25 between 5pm and 6pm this wednesday.

b) 37% of cars in Albuquerque are white. Last wednesday, 2200 cars drove past Lomas on the I-25 between 5pm and 6pm. Use the Normal approximation to the Binomial distribution to determine the probability that more than 800 of these cars were white.

5. **Skewness.** Let X be a random variable with mean μ and standard deviation σ. Then the Skew of X can be defined mathematically as follows.

$$Sk(X) = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{E(X^3) - 3\mu\sigma^2 - \mu^3}{\sigma^3}$$

The Exponential Distribution is *Skewed Right*. Sometimes this is called *Positive Skew*. Show that for $X \sim Exp(\lambda)$ we have $Sk(X) = 2$ (positive) regardless of λ. *Hint: Find* $E(X^3)$ *via Integration by parts, tabular integration or the gamma function.*

6. **Lack of Memory.** The Exponential has a special property called *Lack of Memory* (the geometric distribution and Professor Halfbrain also have this property). Assuming that $s < t$, this property is defined mathematically as follows:

$$P(X > t | X > s) = P(X > t - s)$$

Prove that this property holds for $X \sim Exp(\lambda)$. *Hint: Find the LHS using the definition of conditional probability. Find the RHS using the CDF. Show that they give the same answer.*

7. Walter and Jesse are cooking batches of their famous Blueberry muffins. The time it takes to make a batch of muffins is Exponentially distributed. They also know that, half the time, they can make a batch in less than 41 and a half minutes.

a) What is the probability that a batch of muffins takes between 20 and 90 minutes to produce?

b) Walter wants to know how long it will take to make a batch of muffins in a worst case scenario. Within what time can they be 99% sure that a batch will be completed?

8. Challenge Problem. A powerful feature of the Moment Generating Function is that it uniquely determines the distribution. In this problem, we will look at a simple example which demonstrates the power of the MGF.

a) Suppose X is a random variable with MGF $M_X(t)$ and let a and b be constants. Show that the random variable $Y = aX + b$ has MGF

$$M_Y(t) = e^{bt}M_X(at)$$

b) Suppose $X \sim N(0, 1)$. Find the MGF of X using the definition.

c) We know that if $X \sim N(0, 1)$ and $Y = \mu + \sigma X$, then $Y \sim N(\mu, \sigma^2)$. Use parts a and b to find the MGF of Y.

d) Now let $U \sim Exp(\lambda)$. The MGF of U was found in last weeks challenge problem (feel free to check solutions). What is the distribution of $V = aU$?