

Please adhere to the homework rules as given in the Syllabus.

**1. Sample Statistics.** Timothy and Jimothy run competing lemonade stands. Their sales (in dollars) for this past week are given in the following table. **All calculations here should be done by hand.**

Timothy ( $x_i$ )	Jimothy ( $y_i$ )
12	10
13	14
9	9
10	11
6	8
11	14
14	13

- a) Calculate the sample mean, variance and standard deviation for Timothy sales. Do the same for Jimothy.
- b) Calculate the sample correlation between Timothy and Jimothy's sales.
- c) Combine Timothy and Jimothys sales into a single variable (14 observations total now). Find the 5 number summary of these data. Interpret the  $3^{rd}$  quartile.

**2. Guess the Correlation.** Go to [guessthecorrelation.com](http://guessthecorrelation.com). Enter your full name as a username. Play the game until you get at least 50 points. Submit a screenshot showing your high-score and user name for credit. The 3 students with the highest score in the class will receive bonus points! (last years winner scored around 200)

**3. Bonferonni Correction.** The point of this comic is that, often, one can find a "significant" result if you search hard enough. If done incorrectly, this is sometimes known as *data dredging* (or p-hacking, data snooping etc.). Another entertaining comic can be found at [xkcd.com/882](http://xkcd.com/882).



Assume we have  $n$  hypotheses, and for each hypothesis, we can have an independent test  $A_i$ ,  $i = 1, 2, \dots, n$ . If a hypothesis is wrong, the test  $A_i$  can give a false positive with probability  $\alpha$  (usually  $\alpha$  is small, like 0.05 or 0.01.). For this problem, let's assume that ALL of the hypotheses are wrong, therefore  $P(A_i) = \alpha$  for  $i = 1, 2, \dots, n$ .

a) What is the probability that at least 1 test gives a (false) positive result? More importantly, what happens as the number of hypotheses gets large?

Hint:  $P(\text{At least one } A \text{ occurs}) = P(A_1 \cup A_2 \cup \dots \cup A_n)$

b) Bonferonni's Inequality states that

$$P(E_1 \cup E_2 \cup \dots \cup E_n) \leq P(E_1) + P(E_2) + \dots + P(E_n)$$

Prove this inequality for two events, i.e.  $n = 2$ . *For challenge points, prove the general version of the inequality using induction.*

c) Now, instead of testing each hypothesis with test  $A_i$ , let's use a new test  $\tilde{A}_i$  which has false positive rate of  $\alpha/n$ . *Comment: In reducing the false positive rate, we almost certainly will increase the false negative rate. No such thing as free lunch.* Use Bonferonni's inequality to show that the probability of getting any false positives is now less than  $\alpha$ .

**4. Data Analysis.** The CDI dataset contains demographic information on the 440 largest counties in the US. The dataset can be read into R (from the course webpage) with the following command.

```
read.csv('http://math.unm.edu/~knrumsey/classes/spring17/MiniProjects/data.csv')
```

Extract the column named "PerCapitaIncome" and divide it by 1000 so the units are \$1000. Your answer to this problem should be in the form of a typed report.

- a) Create a Histogram of the data. Comment on the skew of the data. Does the data look normally distributed? (*Note: I recommend using the argument `breaks=12` to get a better looking histogram*)
- b) Create a Boxplot of the data.
- c) Report the mean, variance, and standard deviation. Also give the five number summary of the data and the IQR.
- d) Give the mode of the data. Do this again after rounding the data to 1 decimal place.
- e) Use a Kernel Density Estimate (KDE) to estimate the mode of the data. Which of the three estimates of the Mode seems to fit the data the best?
- f) Create a QQ-plot of the data. What does this say about the Normality of the data?
- g) Use the  $1.5 \times IQR$  rule to identify potential outliers.

#### Challenge Problems

- h) Create a Box-Cox plot for the data. Choose a transformation of the data based on the Box-Cox plot. Make histograms and QQ-plots to determine the Normality of the transformed data.
- i) Now that the transformed data is (theoretically) more Normal, use the  $z$ -score method on the transformed data to identify potential outliers.
- j) Repeat the process of outlier identification using a Bonferonni correction. Do the results differ?