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This exam contains 4 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Show all of your work, and round all answers to 2 decimal places.

1. (15 points) Timothy, Jimothy, Kimothy and Bob are students in a small seminar course. Their professor decides to choose two of them to interview about the course. To avoid unfairness, the choice will be made by drawing two names from a hat.
  - (a) Write down all possible choices of two of the four names. This is the sample space.
  
  - (b) The random drawing makes all choices equally likely. What is the probability of each choice?
  
  - (c) What is the probability that Bob is chosen?
  
  - (d) Timothy, Jimothy and Bob liked the course. Kimothy did not like the course. What is the probability that both people selected liked the course?
  
  - (e) Suppose now that there are 26 students in the course and 20 of them like the course. If the professor randomly selects a sample of 10 students, what is the probability that 7 of them like the course?

2. I only follow 3 people on Twitter. Suppose that half of the tweets on my feed are from Kimothy, and that the remaining tweets on my feed are split evenly between Timothy and Jimothy.
- Half of Kimothy's tweets are political.
  - All of Timothy's tweets are political.
  - 10% of Jimothy's tweets are political

This morning, I read a tweet at random from my feed, and it was political. What is the probability that it is from Timothy?

3. Consider the following function  $f(x)$ .

$$f(x) = \begin{cases} c \cdot \sqrt{x}, & x = 1, 4, 9, 16 \\ 0, & \textit{otherwise} \end{cases}$$

- (a) What value of  $c$  makes this a valid probability mass function?

- (b) Find  $P(X > 8)$ .

- (c) Find  $Var(X)$ .

- (d) Find  $E(300 - 2X^2)$ .

## 4. Probability Distributions

*Fred catches a trout on approximately 1 out of 5 casts. Let  $X$  be the number of times he will cast until he catches his first trout.*

- (a) What is the distribution (including parameter values) of  $X$ .
  
  
  
  
  
  
  
  
  
  
- (b) What is the mean and variance of  $X$ ?
  
  
  
  
  
  
  
  
  
  
- (c) What is the probability that Fred will have to cast no more than 9 times?

*One out of every ten trout that Fred the Fisherman catches is a Rainbow Trout. Fred catches 12 trout. Let  $X$  be the number of Rainbow Trout that Fred catches.*

- (a) What is the distribution (including parameter values) of  $X$ .
  
  
  
  
  
  
  
  
  
  
- (b) What is the mean and variance of  $X$ ?
  
  
  
  
  
  
  
  
  
  
- (c) What is the probability that Fred catches more than 2 rainbow trout?

*Fred will go fishing for an hour. Assume that the assumptions for the Poisson distribution are met. On average Fred catches 0.25 fish per minute. Let  $X$  be the number of fish that he catches in an hour.*

- (a) What is the distribution (including parameter values) of  $X$ ?
  
  
  
  
  
  
  
  
  
  
- (b) What is the mean and variance of  $X$ ?
  
  
  
  
  
  
  
  
  
  
- (c) Use a Normal approximation to estimate the probability that Fred catches at most 25 fish in an hour.

5. Do question 2 from the STAT 145 Sample Exam 1 from Fall 2009. (Click the webpage link for Tables of Distributions)
6. Let  $X$  be a (continuous) Uniform RV with  $a = 2$  and  $b = 8$ . Recall that the density function of  $X$  is

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

- (a) Derive the expected value and variance of  $X$ .

- (b) Find the CDF of  $X$ .

- (c) Let  $Y = \frac{X-2}{6}$ . First, find the CDF of  $Y$  and then find the PDF of  $Y$ . What is the distribution of  $Y$ ?