STAT 345 Fall 2018
Homework 1 - Set Theory and Counting Name: ______________________________

Please adhere to the homework rules as given in the Syllabus.

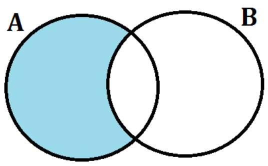
1. Sample Spaces. For each of the following experiments, (1) give the Sample Space using set notation, (2) determine whether each sample space is *discrete* or *continuous* and (3) give an example of a non-trivial event..

a) Experiment: Draw a card at random from a standard deck.
Outcome: The suit of the chosen card.

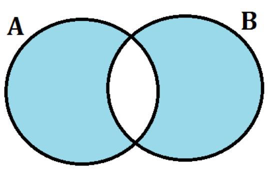
b) Experiment: Alf uses the mobile dating app Tinder for 1 week.
Outcome: The number of matches Alf gets.

c) Experiment: Michael Phelps swims the 200m Butterfly.
Outcome: The time it takes him to finish.

2. Set Operations. a) The difference (subtraction) operator $A - B$ reads "A minus B" is illustrated in the venn diagram below. Give an equivalent expression for $A - B$ using only unions, intersections and complements.



b) The symmetric difference operator $A \triangle B$ is illustrated in the venn diagram below. Note that this is similar to the logical idea of an *exclusive or*. Give an equivalent expression for $A \triangle B$ using only unions, intersections and complements.



3. Venn Diagrams! For each part below, create a 3-set Venn Diagram (with a border representing S) and shade in the region corresponding to the given event.

a) $(A \cap B)^c$

b) $(A \cup B) \cap C$

c) $(A \cup B \cup C) \cap (A \cap B \cap C)^c$

4. Counting

a) At Timothy's Burrito Shop, customers can build their own burrito. There are 5 meat options, 3 rice options and 12 options for add-ins. The customer may choose 2 meats, 1 rice and 5 add-ins. How many different burrito combinations exist?

b) Jimothy has opened up a competing business across the street, Jimothy's Burrito Bowl Shop. He provides exactly the same options as Timothy, except that customers build their own burrito *bowls*. The important difference now, is that the order in which the ingredients are put into the bowl now makes a difference. How many different burrito bowl combinations exist?

5. Suppose I have a bowl with N skittles, K of which are purple. Assume that the percentage of purple skittles is exactly 18%, so that $p = K/N = 0.18$.

a) I grab a handful of 50 skittles from the bowl. Assuming $N = 100$, find the probability that there are x purple skittles in the handful. Using a computer, plot this probability vs. x for $x = 0, 1, \cdots 18$. What is the probability for $x = 9$ (rounded to 3 decimal places)?

b) What if we assume (incorrectly), that the sampling was done *with* replacement? Find the probability that there are x purple skittles in the handful now? Add these probabilities to your plot from part a). What is the probability for $x = 9$ (rounded to 3 decimal place)? How different are the answers?

c) As the population N becomes very large, the difference between sampling with and without replacement becomes negligible. Convince yourself of this fact by repeating parts a) and b) of this problem using $N = 1000$. (Don't need to show work here, just give final answer and include plots).

6. Professor Halfbrain has 10 books on mathematics, 8 books on chemistry and 7 books on astrology (he's a Gemini). He is packing for vacation, and hastily throws 6 books into his suitcase. What is the probability that he has selected 2 books from each subject?

7. A bit is either a 0 or a 1. A *byte* is a sequence of 8 bits and a *nibble* is a sequence of 4 bits.

a) How many different nibbles are possible? Write out the sample space.

b) What is the probability that a randomly generated nibble has two adjacent 1's?

8. **Challenge Problem.** *A Responsible Party.* There are n people who attend a party. Each guest is asked to place his or her car keys into a bowl at the beginning of the night. The next morning, the host (who is very hungover) gives each guest a key from the bowl at random. What is the probability that none of the guests receive the keys to their own car? *Hint: As you work through this problem, it is sometimes easier to think about things for a fixed number of guests, i.e.* $n = 5$*, and then generalize your result.*

a) How many different ways can the keys be distributed to the guests?

b) Let A_i be the event that the i^{th} guest receives his/her keys. Likewise, the event $A_i \cap A_j \cap A_k$ corresponds to the i^{th}, j^{th} and k^{th} guests each receiving their keys. How many ways can the keys be distributed so that this happens? I.e. find $|A_i \cap A_j \cap A_k|$. Using this, what is $\sum_{i<j<k} |A_i \cap A_j \cap A_k|$?

c) Use the general form of the inclusion-exclusion principle to find $|A_1 \cup A_2 \cup \cdots \cup A_n|$. Next, use the rules of counting to find the probability that none of the guests receive the keys to their own car.

d) Using a computer, plot this probability as a function of n for $n = 2, 3, \cdots 10$. As $n \to \infty$, this probability converges (via Taylor Series) to $e^{-1} \approx .3679$. Add a horizontal line at e^{-1} to see this for yourself.