STAT 345 Spring 2018
Homework 12 - Hypothesis Testing

Name: ________________________________

Please adhere to the homework rules as given in the Syllabus.

1. Hypothesis Testing for the Mean. Your favorite brand of cookies claims that on average each cookie contains 10 chocolate chips. Being the skeptic that you are, you decide to test this claim.

a) Write out the Null and Alternative Hypotheses for each of the following situations.

   i) You seek to find evidence that the average number of chips per cookie is not 10.

   ii) You seek to find evidence that the average number of chips per cookie is less than 10.

   iii) You seek to find evidence that the average number of chips per cookie is more than 10.

b) Suppose you purchase $n = 9$ cookies and compute $\bar{x} = 9$ and $s = 3$. Calculate the test statistic.

c) Compute the p-value for each of the three tests above and make a conclusion. Your conclusion should be in terms of the problem. For each case use $\alpha = 0.05$.

d) Repeat parts b and c for $n = 50$.

2. **Data Analysis**. Researchers hypothesize that there will be more serious crimes (per capita) in counties with high poverty rates than in counties with low poverty rates. Use the CDI dataset on the webpage

```
cdi = read.csv('http://math.unm.edu/~knrumsey/cdi_sample/csv')
```

In 2000 (the year this CDI dataset is from), the average poverty rate was about 10%. For each of the $n = 100$ counties in the dataset, check to see if it the poverty rate is higher than average by typing:

```
high_poverty = (cdi$PercentBelowPoverty > 10)
```

Divide the per capita crime data into two sub-populations using this criteria.

a) State the hypotheses using mathematical notation.

b) Calculate the appropriate 2-sample test statistic.

c) Use the `t.test()` function to find the approximate degrees of freedom for this test.

d) State your conclusion and interpret the results in terms of the problem. Does there seem to be a statistically significant difference? Is the difference practically significant?

3. **Type I and Type II Errors.** Harry has a single observation $X \sim Exp(1/\mu)$ and he wants to consider the following test. *Note: Remember that in this parameterization μ is the mean and the CDF is $F(x) = 1 - e^{-x/\mu}$.*

$$H_0 : \mu = 1 \qquad H_a : \mu = 2$$

Harry will use the following "rejection region". $R = \{x | x > 1.5\}$.

a) Find α, the probability of Type I Error for this test.

b) Find β, the probability of Type II Error for this test. What is the *power* of this test?

c) Harry has decided that he cannot afford to have a Type I error probability which is more than 0.10. Give a new rejection rule which will ensure that $\alpha = 0.10$. What is the power of this new test?

4. Suppose a new drug is in development for reducing Systolic blood pressure. The drug is intended for a population of people with high SBP (say SBP = 135 mmHg). When a member of this population takes this drug, the reduction in SBP is assumed to be Normal with mean μ and sd σ where both parameters are unknown.

Professor Halfbrain, the replacement of recently fired Mr. SS, would like to test whether the drug reduces SBP by a practically significant amount (recall that researchers determined that a reduction of 7 mmHg is deemed important). He proposes to test the hypotheses

$$H_0 : \mu = 7 \quad vs \quad H_a : \mu > 7$$

a) For a sample size of n and a significance level of $\alpha = 0.05$, find the rejection region R for this test, in terms of the sample mean $\bar{x}$ and sample sd S.

b) Suppose that the data is collected ($n = 348$), and the sample mean and sd are computed as $\bar{x} = 8.38$ and $S = 5.85$. Make a conclusion and interpret the results in terms of the problem.

c) Calculate the p-value. It should agree with your conclusion in b. *Note: This is not really necessary since we already found the rejection region, but its good practice!*

d) Power analysis: Lets assume that the true value of σ is 5.85. Find the power of this test if the true value of μ is 7.5. Interpret this result.

e) Keeping σ fixed at 5.85, write out the general form of the power for μ and plot the power as a function of μ for $\mu \in [6, 10]$.

f) Assume that the true value of σ is 5.85 and μ is 8.5 mmHg. Plot the power as a function of n for $n = 5, \cdots 2000$. What sample size is required for a power of 80%?

5. Challenge Problem. *Emperical Testing.* Between Janruary 1st, 2016 and April 9th 2018, NBA player James Harden took 1,764 three pointers. Data can be found on the course webpage at

`math.unm.edu/~knrumsey/classes/fall18/harden.csv`

The data is recorded sequentially (in order) where a 1 represents a made shot and 0 represents a missed shot. Clearly, we are dealing with Bernoulli trials here. Ideally, we would like to analyze this data by assuming that $Y = \sum_{i=1}^{1764} X_i$ is a Binomial random variable. **However,** the Bernoulli trials have to be independent in order for this result to hold. In this case, it is reasonable to assume that the trials may NOT be indepdent (hot or cold streaks etc.). In general, this is a very difficult thing to test, but we can attempt to do so *empirically.*

The Hypothesis: We would like to test,

$$H_0 : X_1, X_2, \cdots X_{1764} \text{ are independent} \qquad H_a : X_1, X_2, \cdots X_{1764} \text{ are not independent}$$

The Test Statistic: We need to come up with a statistic that might be able to test this hypothesis in some way. Here is one possible choice.

$$T = \text{the longest streak of made shots}$$

The Reference Distribution: Although T can be easily computed from the data, it does no good unless we know the dsitribution of T *given that* H_0 *is true.* This is very hard (and maybe impossible) to find this distribution exactly. But we can get an empirical distribution via simulation to use for reference. *If the Null hypothesis is true*, the $X_1, \cdots X_{91}$ are independent Bernoulli random variables. This data can be simulated in R by using `x <- rbinom(91, 1, p)` where p should be set to the proportion of Curry's made shots. If you repeat this process a large number of times (say 1000) and calculate T each time, you will have a reference distribution that you can compare the actual observed value of T to.

Empirical p-value Once you have a reference distribution, you can approximate the p-value in the usual way. Think about how you would do this, and ask your instructor for help if necesarry.

The Problem: Conduct this Hypothesis test, following the steps above. Provide a histogram showing the reference distribution you simulated, and add a vertical line illustrating where the observed value of the test statistic falls. Give the emprical p-value and interpret your results at the 5% significance level.

Calculating T: Here is an R function which will calculate the value of T given a binary vector.

```
longest_make_streak <- function(x){
  i <- 0;   m <- 0;
  for(j in 1:length(x)){
    if(x[j] == 1){ i <- i + 1 }
    else{ i <- 0  }
    if(i > m) m <- i
  }
  return(m)
}
```

Note: You should also look at the longest miss streak which requires a simple change to the function.