STAT 345 Spring 2018
Homework 4 - Discrete Probability Distributions Name: ______________________

Please adhere to the homework rules as given in the Syllabus.

**1.** **Coin Flipping.** Timothy and Jimothy are playing a betting game. Timothy will flip a fair coin until it lands Heads. If $X$ is the total number of flips, then Jimothy has to pay Timothy $g(X)$ dollars, where $g(X)$ is defined as

$$g(X) = \begin{cases} -5, & X = 1 \\ 0, & X = 2, 3, 4 \\ M, & X \geq 5 \end{cases}$$

Using a computer, plot $E(g(X))$ as a function of $M$. For what value of $M$ is the game "fair" in the sense that $E(g(X)) = 0$?

**2.** **Build a Test Problem**. For this problem, try really hard to **be creative**. Try to use an example that is from your life, work or make something up. Your examples can be humorous or not humorous. A bonus point will be given for unique questions.

Come up with a Bernoulli random variable (i.e. a **random** variable which can be either 0 or 1). The random variable should be independently repeatable $n$ times (like flipping a coin). Using your Bernoulli random variable, explain an experiment which leads to a Binomial random variable $U$ and another experiment which leads to a Geometric random variable $V$. Ask 3 or 4 test-type problems and give the answer (you don't have to show your work). If somebody gives a good enough answer here, I might use it on the midterm.

**3.** Professor Halfbrain has just created a fair five sided die, with sides numbered 1 through 5. This die is rolled once, and we let $X$ be the number facing up.

a) Determine the distribution, expected value and the variance of $X$.

Professor Halfbrain constructs two more of these fair five sided die, but he chooses to number these 2 through 6 and 3 through 7 respectively.

b) If the Professor throws all 3 dice, what is the probability that the sum of their numbers is equal to 7? *Hint: Approach this as a counting problem.*

c) All three die are placed inside of a bag. The Professor reaches into the bag, grabs a die at random and tosses it. Let $Y$ be the number facing up. Find $P(Y < 3)$. *Hint: Use Law of Total Probability.*

**4.** JeBron Lames is a basketball player who makes his free throws 70% of the time. Whether or not he makes a free throw is independent of his previous shots. Suppose JeBron shoots 10 freethrows, and let $X$ be the number of free throws that he makes.

a) Identify the distribution of $X$, and write out it's PMF.

b) Determine the expected value and the variance of $X$.

c) Find $P(X = 5)$ and $P(X > 2)$.

**5.** Legendary STAT 345 Instructor Zach Stuart plans to swipe right until he gets a match. The probability of a match is (sadly) quite low at 3%. Let $X$ be the number of times Zach will swipe right.

a) Identify the distribution of $X$, and write out it's PMF **and** it's CDF.

b) Determine the expected value and the variance of $X$.

c) What is the probability that Zach get's a match before he runs out of swipes (100 swipes per day).

**6.** Walter White and Jesse Pinkman run an operation where they cook blue(berry) muffins. Let $X$ be the number of blueberries found in one of their muffins and assume that $X$ follows a Poisson distribution with mean $\lambda$.

a) Assume $\lambda = 9$. One of Walter and Jesse's customers gets very angry when he gets a muffin with 0 blueberries. What is the probability that this happens?

b) Blueberries are in short supply! Walter and Jesse want to reduce the average number of blueberries per muffin ($\lambda$) to save costs. What is the smallest value of $\lambda$ possible, which would still ensure the probability in part *a*) is no more than 0.01?

c) Assume now that the customer gets angry if he gets a muffin with less than or equal to 1 blueberry. For the same requirement in *b*), what is the smallest possible value of $\lambda$ now? (*Note: You will not be able so solve for* $\lambda$ *analytically. Use any method you like (including guess and check) to give a correct answer to* 1 *decimal place).*

d) Assume $\lambda = 9$. Let $Y$ be the number of blueberries found in a dozen (12) of Walter and Jesse's muffins. What is the distribution, mean and variance of $Y$?

**7. Challenge Problem:** Refer to the Bayes Theorem example we did in class, where Alf is being tested for a rare disease. Recall that, even if Alf tests positive the probability he has the disease is fairly small. Therefore Alf may need to take $n$ tests instead of just one. If we let $X$ be the number of times Alf tests positive, we are dealing with a Binomial distribution where the probability of success is conditional on whether or not Alf has the disease. Let $D = \{\text{Alf has the disease}\}$ and assume $P(D) = 0.00001$.

$$X|D \sim Binom(n, 0.99)$$
$$X|D^c \sim Binom(n, 0.05)$$

Now, assume that Alf tests positive for $k$ tests ($k = 0, 1, \cdots n$). We can use Bayes Theorem as follows,

$$P(D|X = k) = \frac{P(X = k|D)P(D)}{P(X = k)}$$

Where $P(X = k) = P(X = k|D)P(D) + P(X = k|D^c)P(D^c)$ by Law of Total Probability.

**a)** Compute $P(X = k|D)$ and $P(X = k|D^c)$ using the appropriate Binomial PMF.

**b)** Use Bayes Theorem to give an explicit expression for $P(D|X = k)$ in terms of $n$ and $k$.

**c)** If Alf takes $n = 4$ tests and tests positive $k = 3$ times, what is the probability he has the disease?

**d)** How many consecutive tests does Alf need to test positive for, before the probability he has the disease is at least 0.99?