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This exam contains 5 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Show all of your work, and round all answers to 2 decimal places.

1. Suppose  $X_1, X_2, \dots, X_n$  are iid with the following density function,

$$f(x) = \frac{1}{2\theta + 1} e^{-x/(2\theta+1)}, \quad x > 0$$

- (a) Find the mean of  $X_i$ .
- (b) Find  $\hat{\theta}$ , the Method of Moments Estimator for  $\theta$ .
- (c) Find the MSE of your estimator in b).

2. (10 points) Let  $X_1, X_2, \dots, X_{100} \stackrel{iid}{\sim} \text{Exponential}(1/3)$ . Assume that  $n = 100$  is large enough so that the CLT applies.

(a) What is the approximate Sampling Distribution of the sample mean  $\bar{X}$ ? *Be specific.*

(b) Approximate  $P(\bar{X} > 0.34)$ .

3. (20 points) The wait time at a certain bank is known to be approximately Normally distributed. A sample of 25 wait times has a mean of 18.5 minutes, and the sample standard deviation is  $s = 3.1$  minutes.

(a) Construct a 99% Confidence Interval for the true mean wait time at this bank. **Interpret your interval.**

(b) Construct a 95% Lower Confidence Bound for the true mean wait time at this bank. **Interpret your interval.**

4. (20 points) Robin Hood is practicing his archery. He shoots 500 arrows, and hits the bullseye 435 times. As always, assume his shots are independent and have the same probability of hitting the bullseye.
- Construct a 97% Confidence Interval for the true proportion of times that Robin Hood hits the bullseye..
  - Interpret your interval in terms of the problem. Do you feel confident asserting that Robin Hood hits the bullseye less than 9 out of 10 times?
  - What would happen to the length of the interval if you change the Confidence Level to 90%?
  - What would happen to the length of the interval if you change the sample size to 1000?
  - How many arrows would Robin Hood need to shoot for a 97% Confidence Interval to have a Margin of Error no greater than 0.02?

5. (15 points) Suppose it is known that the true mean weight of Tigers in the wild is 400 pounds. Researchers consider a random sample of 9 Tigers *in captivity*, and calculate sample mean and standard deviation of 370 and 55 respectively

At the 5% significance level, the researchers hope to answer the question, "Is this evidence that Tigers in captivity weigh less than the known weight of Tigers in the wild? Conduct this hypothesis test. Be sure to write out the hypotheses, calculate the test statistic, make a conclusion (justify) and interpret in terms of the problem.

6. (8 points) Consider a single observation  $X$  with the following distribution

$$f(x) = cx^{1-\theta}x^\theta, \quad 0 < x < 1$$

where  $c$  is the normalizing constant. Suppose we want to test the hypotheses:

$$H_0 : \theta = 0 \quad vs \quad H_a : \theta = 1$$

Consider the rejection region  $R = \{x \mid x > 0.6\}$ .

- a) Find the PDF and CDF of  $X$  assuming that the null hypothesis is true.

- b) Find the probability of Type I Error.

- c) Find the PDF and CDF of  $X$  assuming that the alternative hypothesis is true.

- d) What is the power of this test?

- e) Consider a rejection region  $R = \{x \mid x > k\}$ . Find the constant  $k$  so that the significance level of the test is 0.01. What is the power of this new test? *Use back of page to answer this question.*