STAT 345 Spring 2018
Midterm Review **Solutions** Name: **Solutions**

**1.** We did this one in class.

**2.** Let $A$, $B$ and $C$ be the events that Alf, Betty and Carl are the killers. Let $E$ be the event that poison was used. Let's start by noting the information we are given.

$$P(A) = P(B) = P(C) = 1/3$$

$$P(E|A) = 0.5 \quad P(E|B) = 0.1 \quad P(E|C) = 0.99$$

I am asking you for the probability that Carl is the killer *given that* poison was used, i.e. $P(C|E)$. Using Bayes theorem and the Law of TOtal probability in the denominator we have

$$P(C|E) = \frac{P(E|C)P(C)}{P(E)} = \frac{P(E|C)P(C)}{P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C)} = \frac{0.99(1/3)}{(1/3)(0.5 + 0.1 + 0.99)}$$

And we find that $P(C|E) = 0.6226$. Although this evidence gives us some new information, it is not enough to convict Carl (in my humble opinion).

**3.**
**a)**

$$\begin{aligned} P(B|C^c) &= \frac{P(B \cap C^c)}{P(C^c)} && \text{definition of conditional prob} \\ &= \frac{P(B \cap C^c)}{1 - P(C)} && \text{complementation} \\ &= \frac{0.15}{1 - 0.4} = 0.25 \end{aligned}$$

**b)** By law of total probability,

$$P(B) = P(B|C)P(C) + P(B|C^c)P(C^c) = 0.5(0.4) + 0.25(0.6) = 0.35$$

**c)** We use independence here: $P(A \cap B) = P(A)P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B) = 0.45 + 0.35 - (0.45)(0.35) = 0.5075$$

**4.**

**a)**

$$E(X) = 1(0.1) + 2(0.5) + 3(0.2) + 5(0.1) + 7(0.1) = 2.9$$
$$E(X^2) = 1^2(0.1) + 2^2(0.5) + 3^2(0.2) + 5^2(0.1) + 7^2(0.1) = 11.3$$
$$Var(X) = E(X^2) - E(X)^2 = 11.3 - 2.9^2 = 2.89$$

**b)**

$$E(\sqrt{X}) = \sqrt{1}(0.1) + \sqrt{2}(0.5) + \sqrt{3}(0.2) + \sqrt{5}(0.1) + \sqrt{7}(0.1) = 1.11$$

**c)**

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.1, & 1 \leq x < 2 \\ 0.6, & 2 \leq x < 3 \\ 0.8, & 3 \leq x < 5 \\ 0.9, & 5 \leq x < 7 \\ 1, & x \geq 7 \end{cases}$$

**5.** We did this one in class too. Also similar to a previous homework question for which the solution is online.

**6.** We did this one in class, but I'll recap it very breifly. $X \sim Binom(50, 1/3)$.

$$E(X) = np = 50/3 = 16.67$$

$$Var(X) = np(1-p) = 100/9 = 11.11$$
$$P(X = 20) = f(20) = (1/3)(2/3)^{19} = .0705$$
$$P(X \leq 20) \approx \Phi(\frac{20 - 16.67}{\sqrt{11.11}}) = 0.8411$$

Note that the exact probability (found with R) is 0.8741.

Note that $Y \sim Geom(1/20)$.

$$E(Y) = 1/p = 20$$
$$Var(Y) = (1-p)/p^2 = (19/20)/(1/20^2) = 380$$
$$P(Y = 10) = (1/20)(19/20)^9 = 0.0315$$
$$P(Y \geq 10) = 1 - P(Y < 10) = 1 - P(Y \leq 9) = 1 - (1 - (19/20)^9) = 0.6302$$

**7.** We did this one in class. Also several similar examples on homework.

## 8.

**a)**

$$P(X > 150) = 1 - P(X \leq 150) = 1 - \Phi(\frac{150 - 140}{20}) = 1 - \Phi(0.5) = 0.3085$$

**b)**

$$P(90 < X < 150) = \Phi(\frac{150 - 140}{20}) - \Phi(\frac{90 - 140}{20}) = 0.6853$$

**c)** They are looking for $z$ such that $\Phi(z) = 0.01$. This gives $z = -2.33$. To convert to mg we use

$$x = \mu + \sigma z = 140 + 20(-2.33) = 93.4mg$$

They should offer a refund if the coffee has less than 93.4 mg of caffeine.