

**1.** Timothy and Jimothy are taking a class with 4 other people (6 students total). The professor will select two different students at random to answer a question on the board.

a) Write out the sample space of all possible selections of two students. How many outcomes are there?

b) What is the probability that both Timothy and Jimothy are selected?

c) What is the probability that either Timothy or Jimothy is selected?

**2.** Alf, Betty and Carl are on trial for murder. Before the evidence is given, Alf, Betty and Carl have equal chances of being the killer. We also know the following,

- If Alf is the killer, there is a 50% chance that he would use poison.
- If Betty is the killer, there is a 10% chance that she would use poison.
- If Carl is the killer, there is a 99% chance that he would use poison.

The autopsy results come in, and we now know that Bob was poisoned. Given this new evidence, what is the probability that Carl is the killer? In your opinion, do we have enough evidence to convict Carl? (This is sort of the idea behind circumstantial evidence).

**3.** Let  $A$ ,  $B$  and  $C$  be events and assume:

- $P(C) = 0.4$
- $P(B \cap C^c) = 0.15$
- $P(B|C) = 0.5$
- $P(A) = 0.45$
- $A$  and  $B$  are disjoint.

a) Find  $P(B|C^c)$

b) Find  $P(B)$

c) Find  $P(A \cup B)$

4. Let  $X$  be a discrete random variable, and let the following table describe its pmf.

$x$	1	2	3	5	7
probability	0.1	0.5	0.2	0.1	0.1

- a) Find the expected value and the variance of  $X$ .

- b) Find  $E(\sqrt{X})$

- c) Find the CDF of  $X$ .

5. Company XYZ produces plastic discs. Suppose that the number of scratches on a disc is a Poisson RV with mean  $\lambda = 1.5$ .

- a) Find the probability that a disc has no scratches on it. Also find the probability that a disc has exactly 1, 2 or 3 scratches on it.

- b) Let  $Y$  be the number of dollars that the company can sell a disc for, and assume

$$Y = \begin{cases} 5, & X = 0 \\ 2, & X = 1, 2, 3 \\ 0.5, & X \geq 4 \end{cases}$$

Find the expected selling price of a plastic disc.

**6.** Fred and George are spending their spring break on their second favorite past time, picking up chicks. (Statistics is their first favorite past time). Fred gets a girls number 1 out of 3 times and George get's a girls number 1 out of 20 times.

Fred asks 50 girls for their phone number. Let  $X$  be the number of phone numbers he gets.

a) What is the mean and variance of  $X$ .

b) What is the probability that Fred gets exactly 20 numbers?

c) Approximate the probability that Fred gets less than or equal to 20 numbers.

George will continue to ask girls for thier phone number until he gets one. Let  $Y$  be the number of girls George will ask.

a) What is the mean and variance of  $Y$ .

b) What is the probability that the 10<sup>th</sup> girl George asks gives him his first phone number?

c) What is the probability that George will have to ask at least 10 girls for their number before getting one.

**7.** Let  $X$  be a continuous random variable with probability density function (note that a more general/useful version of this PDF can be found by shifting and scaling the PDF, but we'll use this version for simplicity).

$$f(x) = \begin{cases} c \sin x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

a) Find the value of  $c$  that makes  $f$  a valid probability density function.

b) Find the expected value and variance of  $X$ . (Use integration by parts or tabular integration)

c) Let  $Y = \frac{X}{\pi} - \frac{1}{2}$ . Find the mean and variance of  $Y$ .

d) Find the CDF of  $X$  and use it to find  $P(X \leq 1)$ .

**8.** A cup of coffee has 140mg of caffeine on average with a standard deviation of 20mg. Assume the distribution of caffeine in a cup of coffee is Normally distributed.

a) What is the probability that a single cup of coffee has more than 150mg?

b) What is the probability that a single cup of coffee has between 90 and 150mg ?

c) A coffee company decides that they will refund a customers money if the coffee doesn't haven't enough caffeine. What should they choose as a cutoff if they want to provide refunds just 1% of the time?

**9.** Let  $X$  be the number of miles that a car can drive before it breaks down. Assume that  $X$  is an exponentially distributed random variable with mean 100,000 (remember that mean is  $E(X) = 1/\lambda$ ).

a) Find the median of  $X$ .

b) Find  $E(X^{2.5})$ . *Hint: Make a  $U$ -substitution and then use the Gamma function.*

c) Find the probability that a car will last more than 200,000 miles.

d) Find the probability that a car will last more than 200,000 miles, given that it has already made it 100,000 miles. *Hint: Use loss of memory property.*