1. The weight of a grasshopper is a random variable X. It is assumed that $X \sim Gamma(2, 1/\theta)$.
   The CDF of X is
   $$F(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/\theta}(1 + x/\theta), & x > 0 \end{cases}$$
   If the rainfall this year has been regular, then we assume that $\theta = 1$. If it has been a dry year, then θ will be considerably smaller. Consider testing the hypotheses

   $$H_0 : \theta = 1 \quad vs \quad H_a : \theta = 0.1$$

   using the rejection region $R = \{x \mid x < 0.5\}$.

   a) Find the probability of Type I Error for this test.

   b) Find the probability of Type II Error for this test.

   c) What is the power of this test?

   d) *Challenge:* If you have extra time, derive the CDF given above starting with the probability density function.